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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

DECEMBER 1919

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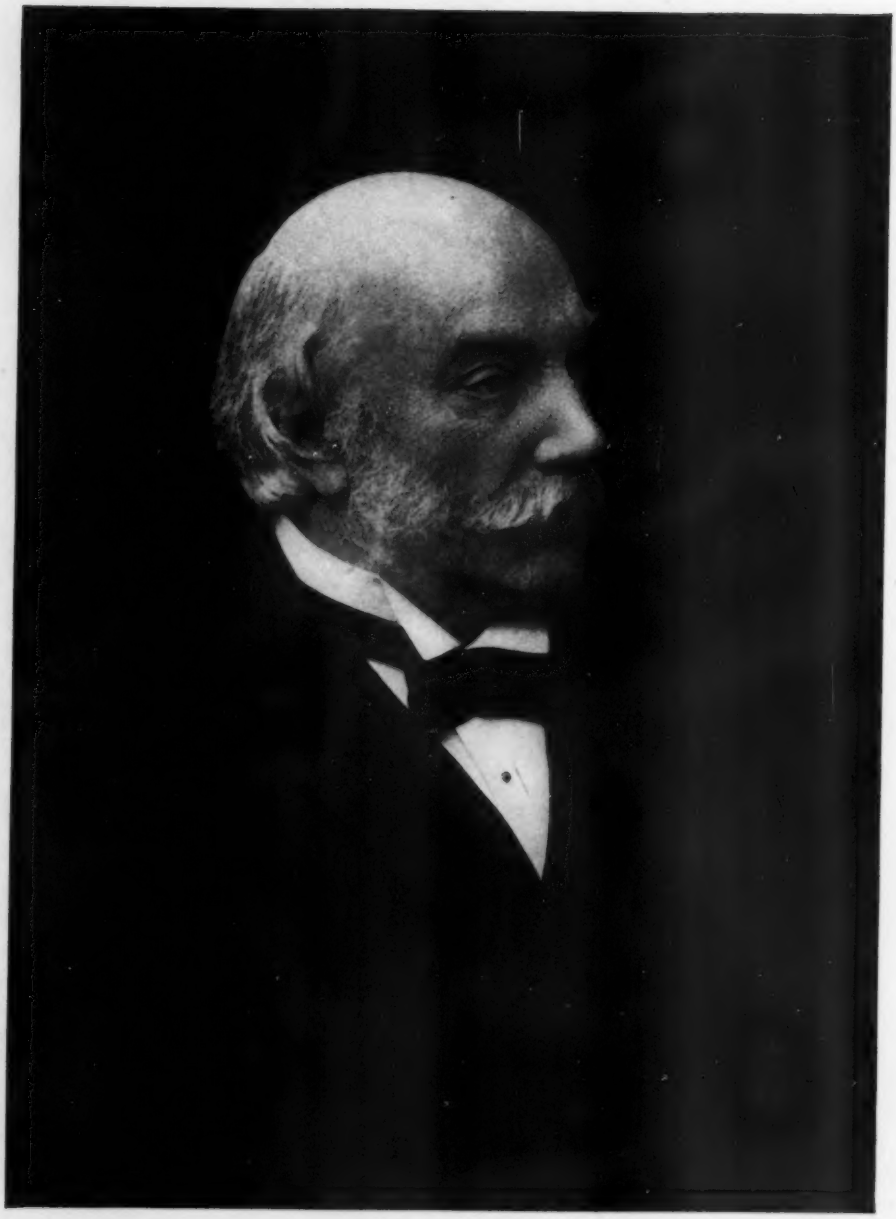

LORD RAYLEIGH
1842–1919

THE
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VOLUME L DECEMBER 1919 NUMBER 5

LORD RAYLEIGH, 1842–1919

By SIR JOSEPH LARMOR and H. F. NEWALL

By the death of Lord Rayleigh on June 30, 1919, the last representative of the great age in theoretical physical science connected with the names of Kelvin, Helmholtz, Stokes, Kirchhoff, Maxwell, has been removed. The novel outlooks opened up by these men into the unfathomable yet intelligible constitution of the material universe have been made secure, though the fruits of their development by the combined labors of the scientific world are far from being yet exhausted. In the last quarter of a century new horizons have opened out, revealing facts and possibilities, in electrical science and in astronomical observation, even more marvelous than the ones which they reduced to intellectual order and system, and made amenable to practical utilization. Their complete comprehension will go yet deeper into the scheme of nature and is still largely a problem for the scientific thought of the future; however novel may be the paths into which it may lead, they will be approached only along the explored highways of the physical science that we have inherited.

John William Strutt, third Baron Rayleigh, was born at Langford Grove, Essex, on November 12, 1842, and succeeded his father in the title in 1873. His early education was frequently

interrupted by delicate health. He was for a very short time at Eton and then for a still shorter time at Harrow; but it was found necessary to send him to the milder climate of Torquay, and he was there put under the care of Mr. G. T. Warner, who had been a classical master at Harrow. In 1861 he went to Trinity College, Cambridge, and apparently notwithstanding this classical bent of his early education he graduated in January 1865 in mathematical science as senior wrangler and first Smith's prizeman. The economist Alfred Marshall was second wrangler in the list.

In 1866 he was elected to a fellowship of his college, which he vacated in 1871 on his marriage with Evelyn, daughter of James Maitland Balfour, of Whittinghame, the sister of Arthur James Balfour, formerly Prime Minister and now Foreign Secretary, and of Francis Maitland Balfour, who in a short life established a great reputation in the science of embryology and the evolution of the forms of life. Of his four sons two survive him: Robert, now professor of physics in the Imperial College of Science and Technology, who succeeds to the title, and Arthur, who was navigating officer on the flagship of the first battle squadron and fought in the Battle of Jutland.

After taking his degree at Cambridge he seems to have retired into a life of study and reflection—mainly in the country, first at Langford Grove, Essex, and, after his succession to the barony in 1873, at the family seat, Terling Place, Witham, Essex—which must have laid secure the foundations for his future eminence in the development of physical ideas. He seems to have made a voyage to Canada in 1867, before the days of swift passage across the Atlantic; for he recalls, in his presidential address to the British Association at Montreal, "the impression made upon me, seventeen years ago, by the wild rapids of the St. Lawrence and the gloomy grandeur of the Saguenay." Later in life, as if this experience had established his confidence in the efficacy of such a voyage as a relaxation and a restorative of vital energy, two other considerable voyages were undertaken by him: one to India in 1897–1898, after his term of office as secretary of the Royal Society, and another around Africa in 1908, after his resignation of the presidency of the Royal Society.

It was in the years following 1869 that he began the publication of important contributions relating to physical science, especially optics and acoustics, in a series of papers which were continued without intermission to the end of his life. They have been gathered together by himself, partly in the treatise on the *Theory of Sound*, published first in 1877–1878 and revised and greatly enlarged in 1894–1895, and mainly in the five volumes of *Scientific Papers*. These volumes, issued in 1899, 1900, 1901, 1902, and 1912 contain the papers, 349 in number, down to 1910; and a sixth volume is now in the press.

His earliest public scientific associations were with the London Mathematical Society, at that time a center of mathematical physics even more than of abstract analysis, as the numerous papers from Maxwell, Rayleigh, Kelvin, W. D. Niven, J. J. Thomson, and others testify. In those early days, at a time when the financial resources of the Society had become inadequate for the support of its publications, he effected insurance against the difficulty by a handsome benefaction, the proceeds of which have ever since been a substantial aid to its activities. His contributions to its *Proceedings* turned largely on hydrodynamics, especially the theory of waves and of eddies and jets, and the causes of frictional resistance generally. A series of papers on general principles in statics and kinetics, which were afterward condensed into some early chapters of the *Theory of Sound*, started from universal theorems like the maximum and minimum theorems of energy connected with the names of Kelvin and Bertrand, and the reciprocal relations developed in the treatment of problems in sound by Helmholtz; and he widely expanded them in fruitful directions. They stimulated in other minds the utilization (cf. *Proc. Lond. Math. Soc.*, 1884) of the principle of least action as the necessary unique source of all general relations, in a manner that was systematically developed, with wider dependance on Hamilton's dynamical scheme, some years later by Helmholtz in his Memoir in *Crelle's Journal* on that principle.

Official connection with the university was resumed in 1876, when Rayleigh served as additional examiner in the Mathematical Tripos, then held in vacation time in January. In those days, now

remote, the examination papers of that Tripos circulated far and wide in many forms; they were immediately reproduced as a full-page supplement to the *Cambridge Chronicle*, the old-established local newspaper of county and university; later they were reprinted on smaller pages as part of the contents of the *University Calendar*, then an unofficial record, and in the *Cambridge University Almanac and Register*. It was through this newspaper sheet that in youthful days in Ireland one of the writers first came into touch with Rayleigh's name and work. Some years previously the complaints of Airy in regard to artificiality and want of utility of the questions, and of Kelvin, Tait, and others with regard to their unduly abstract character, had led to the institution of the additional examiner, who was a senior Cambridge graduate of distinction, invited usually from the outer world and probably more in touch with it, in order to assist the traditional two moderators and two examiners in conducting the examination, then of so great prestige. In this way the services of Maxwell, Kelvin, Tait, Rayleigh, Watson, C. Niven, Hopkinson, W. D. Niven, Greenhill, Darwin, were enlisted in successive years, with very marked influence on mathematical study. The new types of questions that were introduced directed attention to new branches of knowledge, and to unfamiliar and often novel aspects of other branches, and of necessity stimulated new ramifications in the teaching; and it may fairly be said that this result was as much due to the mere presence of the outside element in loosening the chains of tradition that had previously bound their resident colleagues as it was to their own direct efforts. The influence of men such as Kelvin, Maxwell, Rayleigh, provided the degree of guidance that was salutary; giving stimulus without dogmatic revolution, it justified on the whole the position of the Tripos papers as the controlling influence in mathematical study, notwithstanding minor abuses. Nowadays it has become fashionable in some circles to depreciate the office and the services of the public examiner as a dealer in artificially concise problems and elegancies that contrast with the sweeping march of a classical treatise. His function is often thought to be merely to verify in as wooden a manner as possible that the syllabus has been learned. Rayleigh has expressed or

implied his dissent from this view not infrequently; and he included his Tripos questions in the *Scientific Papers* (I, 280–286). So long as a meticulous counting up of marks, in order to evolve a minutely graded order of merit, is not insisted upon, and only broad views are taken, the canvassing of a new problem or a new turn of inquiry beyond what the student has been taught to master and reproduce affords even when unsuccessful a powerful stimulus to the better minds, and where it is combined with guidance beyond the handbooks of the day to the original and authoritative sources of knowledge it has been in fact the most potent incentive to progress and to transcend the grooves of the popular teaching of the time. The difficulty is that the field tends to expand so vastly; but restriction of each student to a narrow section of knowledge is not the true remedy.

On the urgent insistence of his friends, especially Kelvin and Stokes, Rayleigh consented to return to Cambridge in 1879 as Cavendish Professor of experimental physics, to take charge of the laboratory which had been left without a head by the premature and lamented death of its earliest chief, Clerk Maxwell; and he retained the chair for five years until considerations of health intervened. When he had thus succeeded to the control of a formal laboratory, he was not long in throwing himself into those problems of interconnected absolute measurement for which the methodical arrangements appropriate to such an institution are most essential. Our present age of electrical engineering was just discernible on the horizon; and it is not the least of the merits of the British Association for the Advancement of Science that it provided the leverage by which Kelvin and Maxwell and their coadjutors laid firm and exact foundations for the practical development that was to be so rapid. They secured the acceptance of a scientific nomenclature, based on the metric system, throughout the world, itself no slight achievement in the light of other experiences in similar directions; and they carried through the initial stages of experiment to place this unitary system on an absolute foundation by determining the numerical relations interconnecting the units. It was this latter problem that was taken over at the Cavendish Laboratory, and for the five years of Rayleigh's direction

it was, in the hands of himself and his sister-in-law, Mrs. Henry Sidgwick, along with Schuster, J. J. Thomson, Glazebrook, Dodds, Searle, and other colleagues, the recognized center of standard electrical determinations. In time the other nations took their full share of the work; later, on the foundation of the National Physical Laboratory with Glazebrook as director, the operations were naturally transferred there.

In the early years he developed the principle of Stokes, previously adumbrated by Sir W. R. Hamilton, that in a dispersive medium periodic disturbances travel as groups of waves with speed different from that of a simple wave-train. In a discussion (1881) in which Willard Gibbs intervened effectively, he deduced from the phenomena of remote variable stars and of occultations that radiations of all wave-lengths must travel through the celestial spaces at precisely the same speed. It seems to follow that the ether that is postulated as the frame for optical and electrical science must be a structureless medium; moreover, if that is so, and not otherwise, the light from distant stars will advance to the observer without being in time extinguished by scattering, other than may result from molecules or particles of matter obstructing its path. The visibility of stars through a nebula, presumably of vast depth, here invites consideration.

Forty years ago the refined use of the spectroscope was only beginning, and there was much misconception as to construction. It was not unusual in the arrangement of prisms to aim at mere dispersion of the lines in the spectrum, forgetting that, if the images of the lines are good, separation of them can be effected just as well by a magnifying eyepiece. This question was settled in the hands of Lord Rayleigh by a solution strikingly concise and so illuminating as to be now common form. He showed that the dispersion of colors depends solely on the aggregate effective thickness of the glass traversed by the beam of light, more precisely, is determined by the difference of the thicknesses, multiplied each by index of refraction, summed along the two terminal rays of the beam and divided by the breadth at emergence. This principle assumes its simplest form when the incident and emergent rays are in parallel beams. If the emergent beam is narrower than the incident, part

of the chromatic separation is really due to magnification, which is in the inverse ratio of their breadths, and not to dispersion, but every part is included in the Rayleigh rule. If the emergent beam is converging so that the spectrum is already focused, the breadth of the beam is not definite, but is becoming narrower as it proceeds; yet the universality of the rule is maintained, the angular dispersion of the wave-fronts getting larger when estimated at the narrower sections, as in fact must be so in order that the foci for the two colors that are compared should be the same whatever front is considered. The essential thing to aim at is sharp focusing of the detail of each line in turn, which is usually secured by the criterion of minimum deviation, very different from that of maximum dispersion. If that is gained there will be ample margin for as much subsequent magnification as may seem to be necessary.

The reasoning, though now familiar, will bear repetition as an illustration of the thesis that physical theory advances, not by elaborate calculations of algebra, nor by isolated guesses, but by the insight that can concentrate all the essentials in one view, and disentangle at sight the controlling principles. The result rests on the principle that the time of transit, proportional to $\Sigma\mu s$, must be the same for the two terminal rays AP and BQ, passing between the wave-fronts AB and PQ of the beam. When the color of the light is changed the paths of the rays from A to P and from B to Q are slightly changed also, and $\Sigma\mu s$ is no longer the same for both rays between these terminal points, for each sum is altered by $\Sigma\mu\delta s+\Sigma s\delta\mu$. But the definition of a ray as the path of *most rapid progress* (not always of shortest aggregate time) requires that for any small variation of path $\Sigma\mu\delta s$ must vanish. Thus the difference of the times of transit of the two rays for the new color is represented very simply by that of $\Sigma s\delta\mu$ taken along the rays. The wave-front for the new color must be PQ' when Q' is farther on along the ray BQ, so that QQ' multiplied by the final index of refraction is equal to $\Sigma s\delta\mu$ for the ray-path AP minus $\Sigma s\delta\mu$ for the ray-path BQ; and this means that the new front is inclined at the angle stated in words by the Rayleigh rule. Nowadays the other side of the matter has become practically essential: the scale of magnitude of the spectrum must be increased so much before the

rays reach the photographic plate that the size of grain of the emulsion shall not interfere with details of the impression; and to this degree it is not the same whether the spectrum, supposed to be sharp enough, is magnified by the instrument or by an eyepiece, as it would be for observation by direct vision in which the breadth of pupil is not a limiting factor.

The great advances in physical optics in the middle of last century, mainly in French hands but also very notably in those of Airy and Stokes, proceeded chiefly by utilization of interferences and changes of phase. Thus they implied a single source of light or different sources so nearly in unison that they may be regarded as emitting a single train of waves. Rayleigh's earliest note to the London Mathematical Society enforced the necessity of the consideration of independent molecular sources; and this subject, that of natural radiation as distinct from simple optical trains of waves, thus initiated from small beginnings, remained a predominant interest throughout his life. Why is it that in the explanation of solar coronas and halos the amplitudes of the radiation scattered by the various particles, being erratic in direction, do not simply cancel out and leave no result? In the current expositions he found no adequate reason why the particles scatter independently so that the energies are additive, and the mean resultant amplitude therefore not nothing, but definite through only $\sqrt{n}$ times a single amplitude, where n is the (very great) number of particles. This led him into lifelong refined discussions of the probabilities or expectations of different resultant values, when the component vibrations or other events are fortuitous, on which he based himself on the laws of probabilities elucidated by Pascal and James Bernoulli and systematically developed in Laplace's great *Treatise*.

Afterward the earlier resuscitation, and development on similar lines of precise statistical analysis, of the kinetic molecular theory of gases by Maxwell had brought the doctrine of equipartition of energy between the various degrees of freedom of the molecules into the foreground. In the hands of Maxwell and of Boltzmann the proof of equipartition did not stop short of application to any vibrating system whatever, obeying the Hamiltonian dynamical

relations; all its modes of freedom seemed to be of equal value. Yet these freedoms for systems involving continuous vibrating media are infinite in number, so that the thermal energy in such a system should fritter away indefinitely into modes of higher and higher frequency. Maxwell did not live to express his views on the paradox of the universal application of his principle of equipartition when Kelvin initiated and pressed home on many occasions an attack on its general theoretical validity. It was indeed difficult, as in most questions of refined probability, to specify exactly and universally what the theorem was; but of the various special systems propounded by Kelvin as instances of its failure there were none that really fell within the accurate enunciation of the principle. It was owing largely to Rayleigh's luminous and critical expositions, still the best account of the subject, that this result was established; though he repeatedly asserted his conviction that some mode of constraint must be at work, overlooked in the statistics, which really inhibits the energy from passing over into vibrations of the most rapid types. In particular he was suspicious of the introduction of mathematical constraints in the argument; for a geometrical constraint is physically merely an elastic connection whose elasticity is so stiff that its period of free vibration is extremely high, and in consequence it never gets into vibration at all. So also a rigid body is one in which waves of deformation equalize themselves with infinitely great velocity of propagation, and therefore deformations never get started into being. Later on this question of equipartition entered on a new phase from the anomalous values found for specific heats at extremely low temperatures, and also from the form of the experimental law of distribution of intensity among the components of natural radiation. In a brief but most remarkable note Rayleigh indicated what the law of distribution for natural radiation ought to be on an equipartition basis; and, when the coefficient in his expression had been numerically corrected for an oversight by Jeans, it was in precise agreement with experimental fact for very long optical waves, though not of course for visible light. Later the illustrative vibrating system used in his argument, the acoustical one of a rectangular box filled with gas, was replaced by Jeans by the more

relevant electric one, a rectangular region of ether isolated within perfectly conducting walls. Either illustration seems to be valid to the same degree to sustain the result; for it is assumed from considerations of general physical experience, as developed on the Stewart-Kirchhoff line of argument, that there is one unique distribution of energy among the wave-lengths which is universal. His mastery of this subject appeared at various times, as when he briefly refuted a subtle argument that rotatory polarization could introduce into such questions an element at variance with the laws of thermodynamics. But he seems to have maintained complete suspense of judgment in regard to the tentative hypotheses, usually of the type of unitary quantification of energy, by which efforts were made to correlate wide groups of new phenomena, particularly of low-temperature specific heat and of cathode emanations and the various connected types of radiation.

It was perhaps natural for him to proceed, in the early days, from the consideration of halos and coronas to the arresting and brilliant, though familiar, natural phenomenon in which scattering of sunlight by a fortuitous distribution of particles is particularly exhibited, the light of the sky and its azure color. The cause of its polarization, whose limitations were closely investigated by Tyndall in experiments on artificial fogs, had been already sagaciously indicated in a different connection by Stokes as early as 1852. In his first paper of 1871 Rayleigh proceeded, as was inevitable at the time, on the elastic-solid theory of the ether, and considered the scattering of a beam of radiation by obstructing elastic spheres small compared with the wave-length. The effect comes out inversely as the fourth power of the wave-length, as he readily verified in accordance with his custom by dimensional considerations, and this at once explains the predominant blue.

It was many years before he seems to have felt himself free (in 1899) to take the plunge of asserting that the molecules of the atmosphere, notwithstanding the vast number of them in the wave-length, are competent by themselves to scatter the blue light. It may have been on account of a difficulty in conceiving them as compact spherical obstacles. But on the electric theory of propagation of waves of radiation in a gas their influence is very simply

expressed; each molecule is polarized by the electric field and so interacts with the electric flux in the radiation. Expressions for the scattering by the molecules are readily obtained in which, when they are taken to be isotropic, the only quantity that occurs is the index of refraction of the medium constituted by their aggregate. In the analysis of Rayleigh the electric theory is not utilized, probably in order to preserve continuity with his early work; the formula for the index of refraction is obtained in a fundamental and illuminating way by determining the scattering along the way, in which direction the phases are all in unison, so that the scattered vibrations, instead of being independent light, combine with the main beam, introducing only a change of phase which is a measure of the delay in propagation. The very remarkable result emerges that the scattering depends only on the number of the molecules (supposed isotropic and all alike) and the index of refraction of the medium which they constitute. This formula of Rayleigh's ought to be applicable to estimate the number of molecules in a gas, i.e., the constant of Avogadro; and from Italian observations by Majorana of intensity of scattering at high altitudes made under the inspiration of Kelvin, but especially from the remarkable later work of the Smithsonian observers, the results have been found so concordant that this method of sky observations, apparently so remote, has even ranked as one of the ways of estimating the value of the principal absolute constant of chemical science.

There was, however, something lacking. The calculations of index of refraction and of the necessary scattering of the radiation had hitherto been made on the hypothesis that the molecules are isotropic and polarize as if they were spherules of a compact dielectric body. But the electrochemical notion of a molecule separable into two ions would rather be that it is electrically bipolar, after the analogy of a magnet; if so, it could not be isotropic. Yet the nearly complete polarization at high altitudes of the light of the sky seen in directions transverse to the solar rays seemed to afford strong presumption of actual isotropy, which the close verification of the Rayleigh formula for the amount of scattered light made almost compelling. Accordingly, when his son, Professor R. J. Strutt (now Lord Rayleigh), succeeded in

detecting and measuring the light scattered transversely, when a strong beam traversed various gases in his laboratory, feelings were in fact divided between surprise that most gases deviated somewhat from perfect polarization of the scattered light and satisfaction that helium deviated so much as to suggest a linear bipolar structure for its molecule. One of Lord Rayleigh's latest papers took up this side of the subject, that of gases with aeolotropic molecules, and provided the results on which further discussion must turn.

His interests extended to all aspects of meteorology and scenery and the sights of open-air life. There is an important paper, in part expository, on stellar scintillation, a phenomenon from which much can be learned concerning the minute irregular structure of the atmosphere; and on his return from a voyage to South Africa he delivered a delightful discourse at the Royal Institution on the colors of sea and sky and their interplay.

His practical knowledge of chemistry, especially in the direction of photography, as was also the case with Sir John Herschel, was extensive. In a footnote to a paper on another subject he threw out a hint as to the origin of certain sporadic color effects, which was realized independently some years later in Lippmann's definite process for color photography. The same topic developed into a mathematical theory in relation to Stokes's problem of the reflection from planes of multiple twinning in crystals of chlorate of potash; and in the case of sound he experimented with a structure consisting of parallel equidistant sheets of muslin which happens to be the exact analogue of the reflection of X and γ rays by the molecular strata of a crystal. He spent some time in developing the manufacture of copies of ruled diffraction gratings by photography and by taking impressions in soft material, a method which has been more recently so successful in the hands of Ives; but the most valuable results were the considerations on the quality and performance of gratings that the attempts suggested to his mind. More than once he expounded with the simplicity of complete mastery the laws of chemical thermodynamics, at a time when this beautiful subject had not shed the abstruse considerations out of which it emerged; a lecture on the "Dissipation of Energy" and

a paper on "The Energy That Is Dissipated in the Mixing of Gases" (1875) may be cited. He had much share (with Maxwell) in securing due recognition in Europe of the rather difficult work of Willard Gibbs, both on the thermodynamic and on the optical side.

He was greatly interested in the history of physical discovery, and perhaps better read in its original classical sources than any of his contemporaries. He ran many a tilt at current scientific authority in order to claim for discoverers, such as Young, Fraunhofer, Balfour Stewart, and Stokes, what he regarded as the recognition due to them; nothing could be more illuminating for the understanding of the historical development of scientific ideas. During his ten years of administration as secretary of the Royal Society, and in other ways, one of his constant preoccupations was to make sure that work of unknown investigators should not be overlooked, owing to imperfections or obscurities in its presentation; and to the end of his life he was always willing to devote valuable time to the assistance of other workers.

A main feature of Rayleigh's work was his sureness of touch. Even in his degree examination at the university it was remarked by Todhunter, no lenient critic, that his answers were fit to go straight to press for publication. No scientific writer of our time has been fertile on so great a scale, whose work could bear the ordeal of republication to any comparable degree. Hardly a line or an expression has had to be altered, except by way of expansion. In 1899 the Cambridge University Press conveyed to him their willingness to publish a collected edition of his scientific papers, a project which had been urged upon them by men of science; the result was three large but not unwieldy volumes, which appeared in 1901. Two volumes were added subsequently, as material accumulated, and a sixth volume will now complete the collection. As already mentioned, in the early time (1877) the *Theory of Sound* included a condensed summary of such papers, only in part reprinted, as had doubtless been evolved in making preparations for that work. A second edition, still in two volumes but of more than double the size, which appeared in 1894, became even to a greater extent a treatise on the fundamental theory of vibrations in

general as well as of sound in particular. The experimentally more tractable subject of electrical vibrations in circuits, telephonic transmission and the like became prominent, as well as the more complex domain of optical waves, of which the simpler and more definite phenomena of sound afforded in many ways apt guidance and illustration. Though the general dynamical principles that are requisite are all covered, regret must be felt that the same sure hand is not available to expound and forecast the modes and potentialities of that wonder of practical development of the present day, recalling the sudden invention of the telephone, where the emission of free electrons at enormous speed is put under simple and refined control for the delicate and almost timeless operations of electric telegraphy across vast distances of free space. In one respect he did intervene in the early stage, with his usual sagacity; when it was announced that wireless signals had been sent across the Atlantic he briefly inquired how the waves could manage to bend round the curved surface of the earth through so great an angle. Being merely optical waves of great length, they ought to be in analogy with waves of light traveling round the surface of a globe with radius reduced from that of the earth in the same proportion as their lengths. Now that the facts are established completely, even to the extent of reception of wireless messages at the antipodes, there is no difficulty in insisting on the mode of explanation that was at hand from the first, that the upper regions of the atmosphere are so much ionized by solar and other influences as to form a conducting screen that effectually deflects the waves before they can pass far into it and so suffer any great absorption. Incidentally the theory of diffraction of radiation around an obstacle, instead of at a sharp edge, has been greatly developed by himself and others, along with much valuable auxiliary mathematical analysis.

He never had occasion, nor perhaps did he possess the mechanical facilities, to acquire the technique of refined spectroscopy on any large scale, though he laid down the principles on which it has developed. But characteristically, at a time when the minute determination of the wave-lengths of standard lines was demanded, he effected some exact measures with simple homely appliances of

the Fabry type, fixed together as usual with sealing wax, a main object being to show how the most laborious part of the work could be safely circumvented by a sheer guess of the integral part of the numerical result, for which sufficient data were available.

He paid much attention to the corrections of systems of lenses though from the philosophical more than from the workshop side. In particular a systematic study of the von Seidel classification of aberrations may be mentioned, in which he rediscovered the apt and natural transformation of the Hamiltonian optical function from point co-ordinates to direction co-ordinates, which had been utilized by Bruns and by Schwarzschild under the name of the eikonal.

Much of the work on radiation was summarized in articles on optics and on wave theory contributed to the *Encyclopaedia Britannica* in the early eighties, which still remain the most concise and authoritative expositions of the subjects of which they treat.

The systematic determination of the electrical constants at Cambridge has been mentioned. In his presidential address to the Mathematical and Physical Section of the British Association at Bristol in 1882 he expressed a conviction that the next great advance might arise from a closer study of some of the simpler phenomena of chemistry, and announced an intention of undertaking a systematic revision of the densities of the principal gases, including, of course, their departures from the ideal standard laws. Within ten years the discrepancies found between nitrogen drawn from the air and the same gas drawn from combined chemical sources had pointed to a new atmospheric substance, and had gradually revealed argon notwithstanding the wholly unexpected absence of all chemical activity. Not the least part of his pleasure in the result arose probably from finding out that Cavendish, whom he admired almost as intensely as he did Young, had recorded his handling of the substance and given its precise amount, which he could not get rid of in the classical determination of the constitution of air. The way being thus pointed out, further scrutiny of air by the process of freezing out at very low temperatures, and of gaseous occlusions in crystals, gave to his energetic coadjutor Ramsay the discovery of helium and the other inert gases, thus

filling up at a single step many gaps in the system of the chemical elements.

The topics selected above in illustration of his work were chosen from memory. A glance through the volumes of the *Scientific Papers* reveals how inadequately they represent his incessant and many-sided activity. Thus there is the great memoir of 1870 on the *Theory of Resonance*, which extended and simplified Helmholtz' classical results of ten years earlier, and perhaps gave the stimulus to writing the *Theory of Sound*. It also introduced, incidentally, new and beautiful principles of approximate physical calculation, which Maxwell developed further in the second edition of *Electricity and Magnetism*. Rather earlier papers, in elucidation of the abrupt pulses and surges in inductance coils, utilized the magnetic indicator of Joseph Henry, which was afterward to become prominent again as a receiver for ether-waves. His early critical studies on double refraction, inspired by Stokes and followed up by Kelvin, formed the authoritative introduction to the subject for the younger generation. The incompleteness of the polarization by reflection from diamond, and from liquid surfaces, led to theory and experiment on the disturbing influence of thin sheets, such as films of oil on water, which in time rendered a complete account of these perplexing disturbances and of resulting uncertainties as to theory. The capillary phenomena of liquid surfaces and of films occupied his attention all through life and were elucidated in all directions with brilliant sidelights into molecular grouping, both by theory and by the introduction of experimental methods, such as observation of ripples, that evaded the many pitfalls. Inspired, as he afterward seemed to remember, by a Tripos problem proposed by Maxwell years before, he was in the field early (1872) as an expositor of the explanation of anomalous dispersion on the idea then broached afresh by Sellmeier, but glimpsed long previously by Young, that the change in the velocity of light which produces refraction is due to resonance and not to mere inertia of the molecules. He followed Langley's gliding experiments preparatory to a solution of the problem of flight with close interest; and at a time when the brothers Wright had just managed to stay up in the air, he announced to the Royal

Society in a presidential address that the year would be memorable as the one in which the problem of artificial flight had been solved. Later, as chairman of the National Physical Laboratory and especially of its Aeronautics Committee, he pointed the way to extended applications of the theory of physical dimensions to the precise interpretation and utilization of experiments on models, which thereby became the main source of progress in design in that branch of technical science.

In his own country Lord Rayleigh's scientific position was always secure, and the honors to which it led were limited only by his aversion to display and reluctance to divert time from what he must have felt to be his proper work. Consequently he was not very well known abroad except in the later years, though through his friend Lord Kelvin he early mixed in the Helmholtz circle; the *Theory of Sound* had been very cordially welcomed by Helmholtz on its appearance in 1877, in the pages of *Nature*. His main piece of administrative activity outside Cambridge was as secretary of the Royal Society for about ten years, where he left many traces visible to his successors of improvements effected on his initiative. He was lord lieutenant of the county of Essex for about ten years. When the Order of Merit was instituted he was selected as one of the twelve original members. For the last eleven years of his life he was conspicuously in place, by the unanimous suffrage of its members, in the great historical office of Chancellor of the University of Cambridge, thus serving as an ideal head for a seat of learning which has a tradition to maintain beyond the bounds of its own nation.

CAMBRIDGE, ENGLAND
September 1919

THE RIGIDITY OF THE EARTH

By A. A. MICHELSON and HENRY G. GALE

In 1914, in the March number of this *Journal* (**39**, 105, 1914) an account was published of a preliminary experiment to determine the rigidity of the earth. At that time it was announced that the experiment would be repeated, using an interference method. The new arrangements were completed and a new series of observations begun on November 20, 1916, and continued until November 20, 1917. The reduction of the observations was interrupted by the war in the summer of 1917 and could not be resumed until April 1919.

The same pits and pipes on the grounds of the Yerkes Observatory at Williams Bay, Wisconsin, were used as in the preliminary experiment. In that experiment pipes 502 feet long and 6 inches in diameter were placed 6 feet underground. One pipe was laid accurately N–S and the other E–W. The pipes ended in pits 10 feet deep and 8 feet square, walled with concrete. The pipes were carefully leveled, and half filled with water, so that an air space extended from end to end of each pipe, above the water. The pipes ended in air-tight gauges provided with windows through which the changes in water-level could be determined by measuring with microscopes the distance between pointers just below the surface of the water and their totally reflected images.

In the present experiment interferometers replaced the microscopes and pointers. The arrangement of the interferometers, one at each end of each pipe, is shown in Figure 1. The compensating glass serves to seal the pipe. The lower mirror is movable vertically and has also the usual adjustments for regulating the width and orientation of the fringes. The film of water over this mirror is kept thin, usually about 0.5 mm, as the viscosity of the water helps to dampen ripples and minor disturbances. The changing thickness of the water film, due to the tides, caused the shift of fringes. The arrangement for recording the fringes was as follows:

Horizontal fringes were projected by the lens *L* on a narrow vertical slit about 0.2 mm in width. Clockwork drew a moving-picture film behind this slit at the rate of about 2 cm per hour. In order to prevent the condensation of moisture on the optical parts the end of the pipe, interferometer, and camera were all inclosed in a galvanized iron box, in which large trays of calcium chloride were kept. An incandescent light was continually burning near the interferometer to keep the temperature slightly raised.

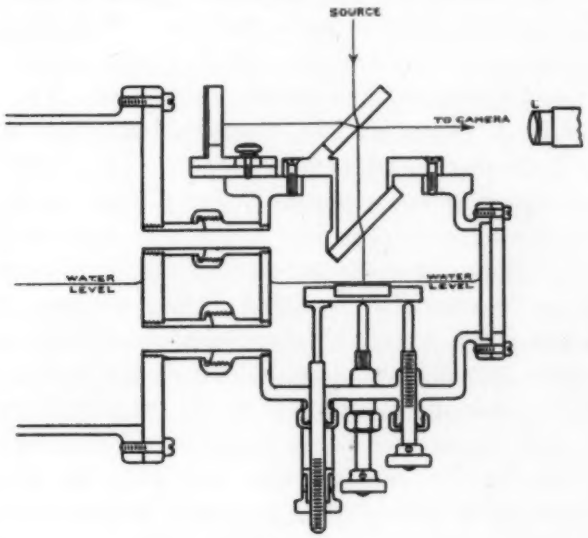


FIG. 1

Other interference arrangements are obvious which would give a displacement of a greater number of fringes, or permit the use of a shorter pipe; e.g., the fringes formed between the water surface and the lower mirror might be used, or the lower mirror might be dispensed with and use made of the fringes formed by the light reflected from the water surface and the vertical mirror. But the arrangement actually used was the most satisfactory, since the long pipes, 502 feet, were already installed.

The sources of light were commercial alternating-current Cooper-Hewitt mercury lamps. They proved very reliable and satisfactory. By using as filters thin films of a saturated solution of

esculin in water, all wave-lengths from the arc shorter than λ 4358 were absorbed, and the positive film used was not sensitive to the longer wave-lengths. The exposed portions of the films were removed and developed each week. The light was abundantly strong for satisfactory negatives, and it was possible to use 1.5 mm diaphragms on the projecting lenses. This gave sufficient sharpness to the fringes, even when there was a considerable change in their focus. It was necessary to readjust and refocus the fringes in only one pit during the entire year, although the width of the fringes was altered once or twice in two other pits. One of the mirrors required resilvering. One of the pits ran throughout the year without readjustment of the fringes or camera. The pits and cameras were in charge of Mr. George Monk and Mr. Frank Sullivan, of the Yerkes Observatory staff.

A relay which moved a shutter in front of the projecting lens was placed in each pit. The four relays were connected in series with a clock in the observatory, so that the time could be conveniently and accurately controlled. Once an hour the clock made contact, and a storage-battery circuit was closed through the relays and the light was cut off by the shutters for about 20 seconds. Very accurate time-marks were secured in this way. The control clock was kept six minutes faster than Central Standard Time in order to simplify the computations and bring the observations into agreement with them. (The longitude of Yerkes Observatory from Greenwich is $5^h\ 54^m\ 13^s$.)

The films were measured by sliding them on a lathe-bed beneath a low-powered microscope. The fringes, estimated to tenths, were counted as they moved up and down, and the numbers recorded for each hour. The difference in the motion at the two ends of each pipe gave the numbers for plotting the observed tides.

The calculated tides were drawn from the computed shift in fringes, the calculations being made for two-hour intervals. The calculations were made under the direction of Professor F. R. Moulton by Mr. Albert Barnett and Mr. Horace Olsen. The formulae are given in the accompanying article by Professor Moulton, "Theory of Tides in Pipes on a Rigid Earth." The value of μ for the water used was found to be 1.3408 for λ 4358, and this is

probably correct to within considerably less than 0.1 per cent for the range of temperatures used.

Calculated and observed curves for the period from March 24 to April 21, 1917, are reproduced in Figures 2 and 3. The dotted curve represents the observed and the full curve (displaced vertically to avoid overlapping) 0.7 of the calculated values of the tides. The ordinates are numbers of fringes, $N=\frac{2(\mu-1)d}{\lambda}$, and one fringe corresponds to 1/1564 mm.

The observed and calculated curves were plotted on long rolls of co-ordinate paper to the following scale: abscissae, 1 cm = 1 hour; ordinates, 1 cm = 2 fringes. In order to have the amplitudes approximately equal, 0.7 of the calculated values were plotted instead of the full amplitudes. Beginning with 10:00 A.M. November 20, 1916, the curves, both observed and calculated, were divided into periods of $12^h.42$ for the semi-diurnal and $25^h.82$ for the diurnal lunar tides. The principal solar tide, period twelve hours, was started at noon of the same day. In order to avoid a cumulative error in the case of the semi-diurnal lunar tide the period $12^h.4206013$ was put on a computing machine and added repeatedly to the initial time to get the exact beginning of each new period throughout the year. This process was repeated, using the period $25^h.8193409$ for the diurnal lunar tide.

The observations were reduced in groups of about a lunar month each, by dividing each period into ten equal parts (twelve in the case of the solar tide), and taking the mean of the first, second, third, etc., ordinates. The resulting values were plotted and any error in computation was usually indicated by the failure of a point to fall on a smooth curve. It is important to treat the observed and calculated tides both in the same way, as any distortion in the resulting sine curves due to lack of complete elimination of other periods affects the two alike. This is, of course, most noticeable in the case of the diurnal tide on account of its smaller amplitude and the smaller number of periods. Mr. Fred Pearson gave valuable assistance in measuring the films, in plotting the curves, and in deducing the various tides from the curves.

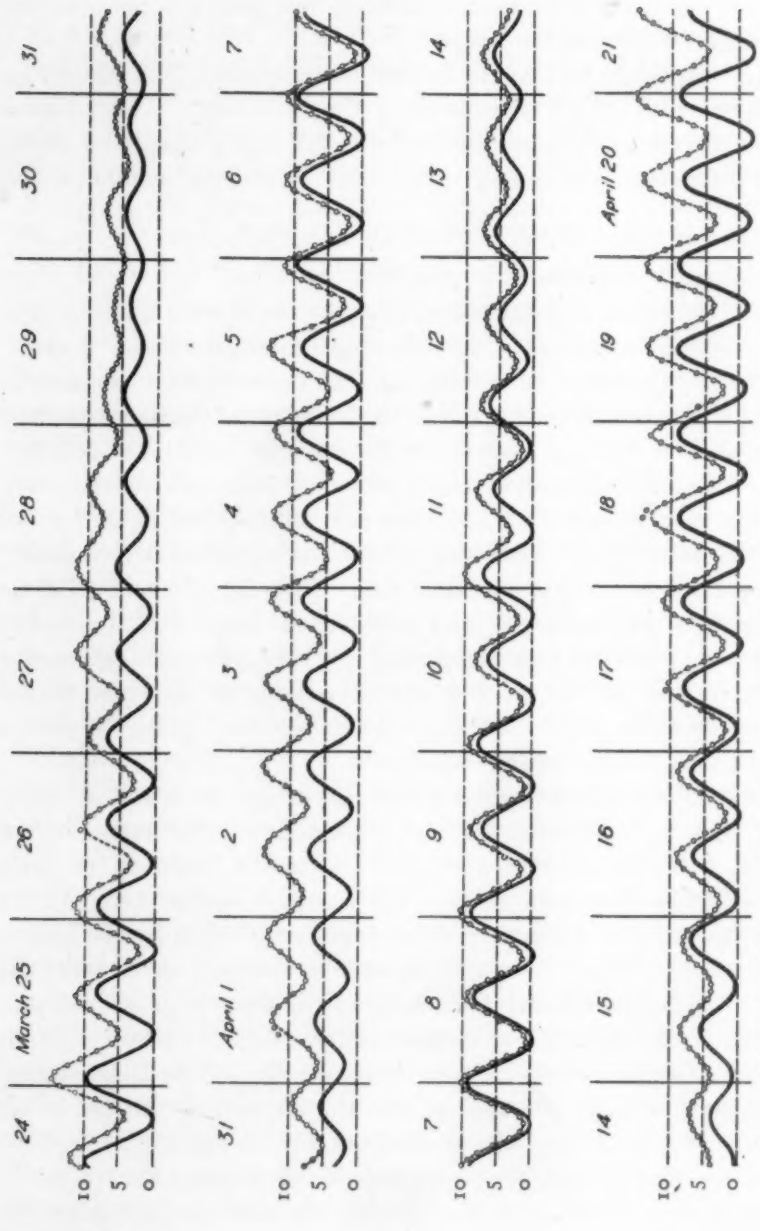


FIG. 2.—N–S tides, March 24 to April 21, 1917. Dotted curves, observed values. Full curves, 0.7 of calculated values

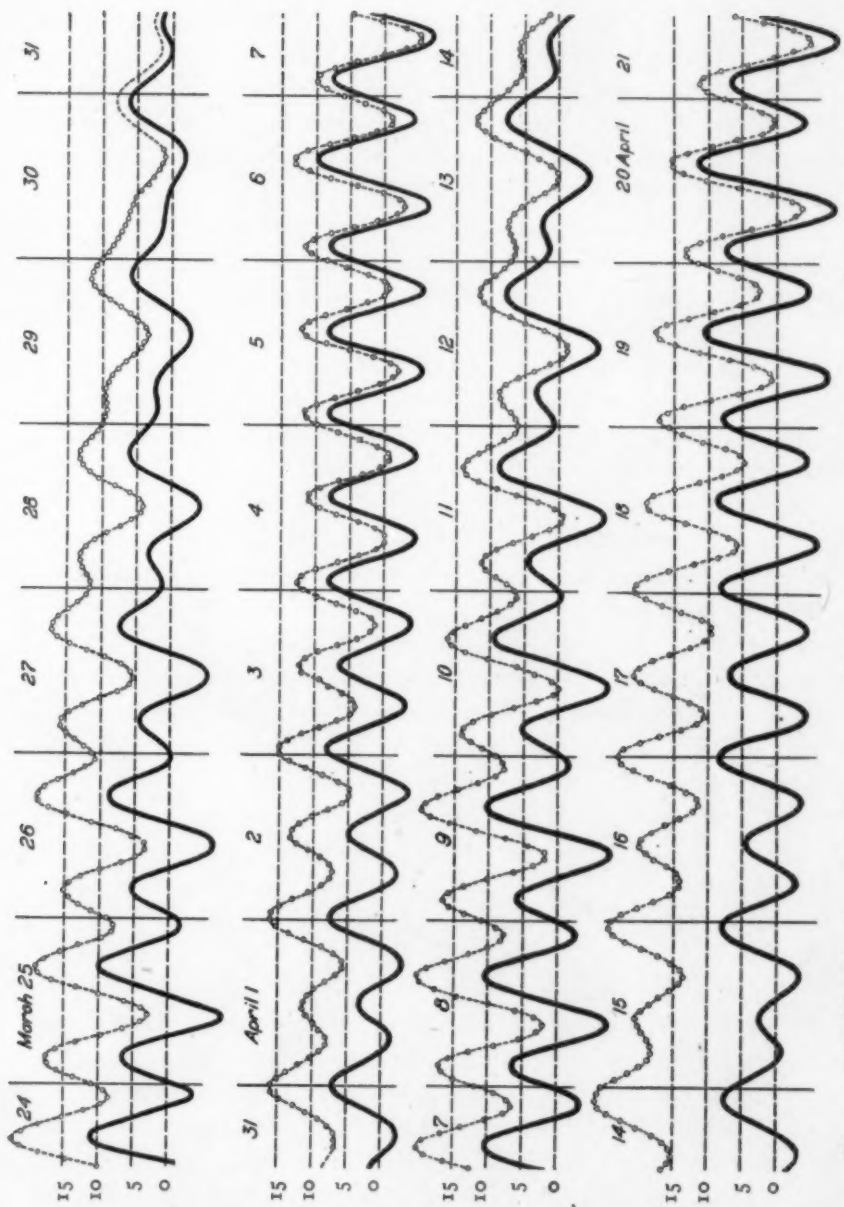


FIG. 3.—E–W tides, March 24 to April 21, 1917. Dotted curves, observed values. Full curves, 0.7 of calculated values

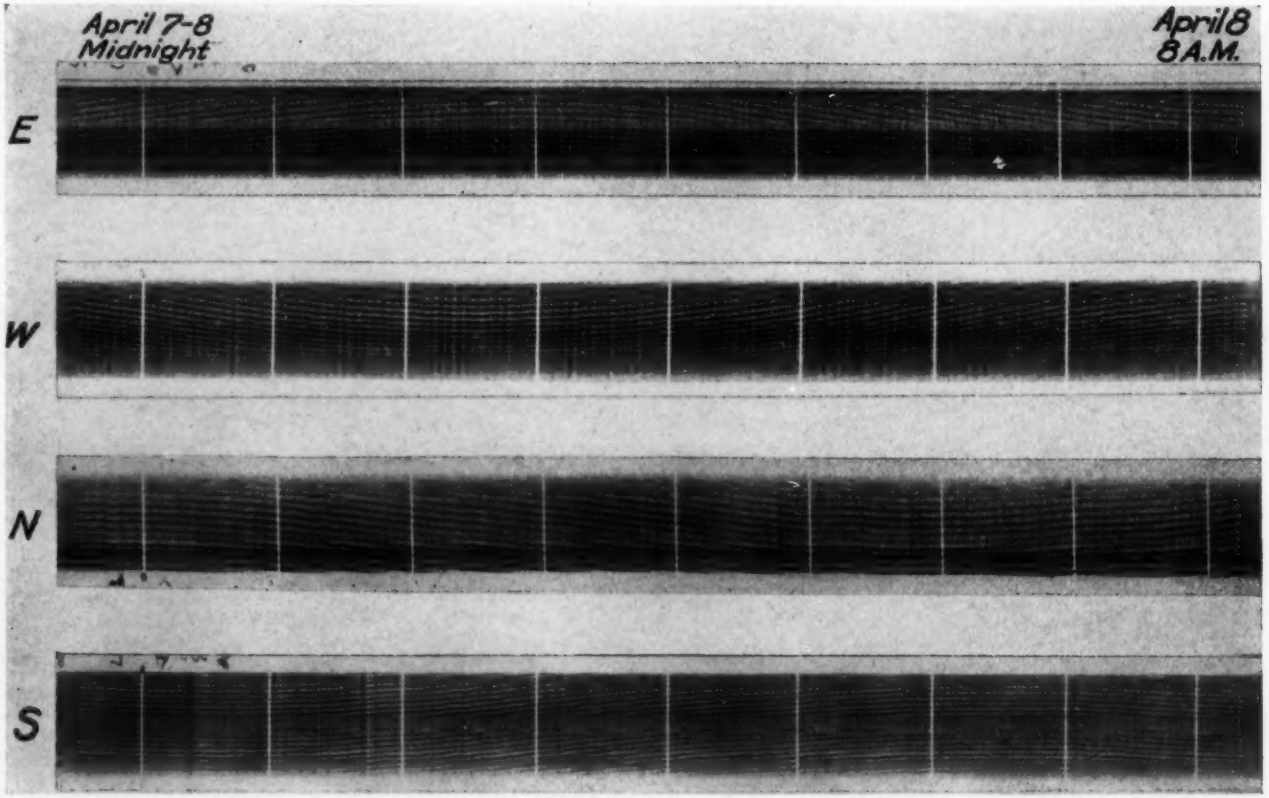
Very little trouble was caused by sudden erratic changes in the fringes. Occasionally, however, earthquakes would cause the fringes to disappear for from ten minutes to half an hour. Once the effects of an earthquake were evident for about six hours. During three hours of this time the fringes were completely obliterated.

The most serious disturbance was a gradual change in the slope of the observed curves. This would often be fairly uniform and gradual for a month or two. At some times the curves would rise and at others fall. Sometimes the N–S and E–W slopes had the same sign, and sometimes opposite signs. We have been able to discover nothing systematic about this drifting. It may have been caused by unequal settling at the ends of the pipes, by temperature changes in the pits, or by tilting in the earth's strata. There were always large shifts of the fringes when the lights came on after having been interrupted by the power company for a half-hour or so. The change of slope was eliminated in reducing each monthly tide, as given in Tables I–VI, and the tide for the whole year, opposite Y in the tables, by distributing the change of level uniformly throughout the period. This change of slope is quite conspicuous in Figure 3, where the observed E–W tide showed a fairly uniform and distinct downward trend throughout nearly the whole month. The change in slope of the N–S curves for the same period is comparatively small, as shown in Figure 2.

Plate XII is from four photographs taken simultaneously in the four pits. The reproductions are positives on the same scale as the originals, and represent fairly well the average quality of the films.

A graphical solution is excellent for detecting erroneous points and serves well to give the ratio of the observed to the calculated amplitude, but for determining the phase-difference of the two curves it is not so satisfactory. A least-squares method was therefore used to secure the ratio of amplitudes, R, and the displacement in phase $\Delta\phi$ of the observed with respect to the calculated tide. The following example illustrates the method of reduction. It is for the semi-diurnal tide, first month, E–W. For convenience the solar periods were divided into twelve parts instead of ten, but in other respects the method of reduction is the same.

PLATE XII



PHOTOGRAPHIC RECORDS TAKEN SIMULTANEOUSLY

FIRST MONTH, SEMI-DIURNAL, E–W OBSERVED

θ	y	Sin θ	y Sin θ	Cos θ	y Cos θ
0°	17.58	0.0000	0.0000	1.0000	17.5800
36	18.74	.5878	11.0153	0.8090	15.1606
72	15.73	.9511	14.9608	.3090	4.8605
108	10.46	.9511	9.9485	− .3090	−3.2321
144	4.83	.5878	2.8390	− .8090	−3.9074
180	0.60	.0000	0.0000	−1.0000	−0.6000
216	−0.20	− .5878	0.1234	−0.8090	0.1698
252	2.58	− .9511	−2.4538	− .3090	−0.7972
288	7.76	− .9511	−7.3805	.3090	2.3978
324	13.48	− .5878	−7.9235	.8090	10.9053
Total			+21.1292		+42.5373

$$a=\frac{21.1292}{5}=+4.2258 \qquad \beta=\frac{42.5373}{5}=+8.5075$$

$$\tan \phi_c=\frac{\beta}{a}=2.0132$$

$$\phi_c=63°35'$$

$$c=\sqrt{a^2+\beta^2}$$

$$c=\sqrt{90.2350}$$

$$c=9.499$$

$$y=9.499 \sin (\theta+63°35')$$

FIRST MONTH, SEMI-DIURNAL, E–W CALCULATED

θ	y	Sin θ	y Sin θ	Cos θ	y Cos θ
0°	17.38	0.0000	0.0000	1.0000	17.3800
36	18.89	.5878	11.1035	0.8090	15.2820
72	16.73	.9511	15.9119	.3090	5.1695
108	11.58	.9511	11.0137	− .3090	−3.5782
144	6.13	.5878	3.6032	− .8090	−4.9591
180	0.90	.0000	.0000	−1.0000	−0.9000
216	0.00	− .5878	.0000	−0.8090	.0000
252	2.23	− .9511	−2.1209	− .3090	− .6890
288	7.16	− .9511	−6.8098	.3090	2.2124
324	12.91	− .5878	−7.5885	.8090	10.4441
Total			+25.1131		+40.3617

$$a=\frac{25.1131}{5}=+5.0226 \qquad \beta=\frac{40.3617}{5}=+8.0723$$

$$\tan \phi_d=\frac{\beta}{a}=+1.6072$$

$$\phi_d=58°7'$$

$$d=\sqrt{a^2+\beta^2}$$

$$d=\sqrt{90.3885}$$

$$d=9.507$$

$$y=9.507 \sin (\theta+58°7')$$

$$R=\frac{c}{d}\frac{0.7\times 9.499}{9.507}=0.699 \qquad \Delta\phi=\phi_c-\phi_d=+5°28'$$

In Tables I–VI the numbers in the columns under M denote the different approximate lunar months; under N is given the

TABLE I

N–S, Semi-Diurnal, Lunar

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1.....	60	6.217	6.373	149°14′	147°48′	0.683	1°26′
2.....	54	5.703	6.489	147 53	144 4	.615	3 49
3.....	48	7.261	6.780	146 15	145 56	.750	0 19
4.....	45	6.750	6.739	148 44	146 23	.701	2 7
5.....	55	6.041	6.269	148 37	148 54	.675	−0 17
6.....	54	6.426	6.551	148 35	147 57	.687	0 38
7.....	54	6.332	6.344	148 40	145 56	.699	2 44
8.....	51	6.052	6.344	147 47	146 58	.668	0 49
9.....	55	6.036	6.319	146 36	147 16	.669	−0 40
10.....	54	5.821	6.210	149 27	147 32	.656	1 55
11.....	54	5.875	6.140	148 14	148 16	.670	−0 2
12.....	55	6.027	6.284	149 18	147 46	.671	1 32
13.....	45	6.330	6.515	152 39	150 36	0.680	2 3
Av....	.....	6.203	.....	.....	.....	0.678 ±.019	1°15′ ±1 2
Y	684	6.173	6.401	148°24′	147°32′	0.675	0 52

TABLE II

E–W, Semi-Diurnal, Lunar

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1.....	60	9.499	9.507	63°35′	58° 7′	0.699	5°28′
2.....	54	8.908	9.096	64 40	58 5	.686	6 35
3.....	48	9.899	10.075	61 28	55 47	.688	5 41
4.....	43	10.220	10.369	69 46	65 2	.690	4 44
5.....	56	9.131	9.380	63 43	57 0	.681	6 43
6.....	53	9.660	9.667	65 4	57 54	.700	7 10
7.....	54	9.496	9.632	64 26	57 6	.690	7 20
8.....	51	9.445	9.331	60 48	55 4	.709	5 44
9.....	55	9.206	9.316	63 24	57 30	.692	5 54
10.....	54	8.773	9.095	64 52	56 32	.675	8 20
11.....	54	8.982	9.108	61 31	57 18	.690	4 13
12.....	55	9.138	9.337	63 12	58 12	.685	5 0
13.....	45	9.809	9.747	67 20	61 44	0.704	5 36
Av....	.....	9.373	.....	.....	.....	0.691 ±.007	6° 4′ ± 55
Y	682	9.380	9.485	63°56′	58°10′	0.692	5 46

number of periods used. For the solar tides missing portions of the observed curves were sketched in, following the computed tides,

thus giving a total of 730 periods for the year, but for the semi-diurnal and diurnal lunar tides only such portions of the observed

TABLE III

N–S SOLAR

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1.....	56	2.554	2.628	104° 8′	99°39′	0.680	4°29′
2.....	62	2.392	2.294	100 13	85 45	.730	14 28
3.....	48	4.883	4.272	93 34	82 59	.800	10 35
4.....	70	2.778	3.132	88 41	80 35	.621	8 6
5.....	58	3.840	3.744	100 26	97 14	.718	3 12
6.....	54	3.595	3.485	102 43	99 59	.722	2 43
7.....	56	2.818	2.570	109 29	103 27	.768	6 2
8.....	54	2.513	2.312	109 1	99 49	.761	9 12
9.....	56	2.771	2.506	87 23	84 18	.774	3 5
10.....	56	2.944	3.108	89 35	86 54	.663	2 41
11.....	56	3.562	3.456	99 10	95 51	.721	3 19
12.....	56	3.154	3.291	108 6	103 40	.671	4 26
13.....	48	3.775	3.889	98 55	99 38	0.679	−0 43
Av....	.....	3.161	.....	.....	.....	0.714 ± .041	5°38′ ±3 16
Y	730	3.140	3.063	99°25′	93°20′	0.718	6 5

TABLE IV

E–W SOLAR

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1.....	56	3.641	3.838	13°54′	9°40′	0.664	4°14′
2.....	62	3.218	3.400	3 31	355 40	.663	7 51
3.....	48	6.172	6.488	358 28	352 59	.666	5 29
4.....	70	4.444	4.665	354 5	350 14	.667	3 51
5.....	58	5.534	5.480	11 19	6 33	.707	4 46
6.....	54	5.104	5.173	16 52	10 59	.691	5 53
7.....	56	3.687	3.731	21 32	13 53	.692	7 39
8.....	54	3.487	3.407	15 3	8 22	.716	6 41
9.....	56	3.450	3.689	359 50	353 28	.655	6 22
10.....	56	4.371	4.595	3 14	356 36	.666	6 38
11.....	56	5.045	4.993	7 57	4 32	.707	3 25
12.....	56	4.689	4.858	16 28	14 16	.676	2 12
13.....	48	5.621	5.626	10 0	6 11	0.699	3 49
Av....	.....	4.459	.....	.....	.....	0.681 ± .018	5°19′ ±1 28
Y	730	4.399	4.522	8° 6′	2°23′	0.681	5 43

curves were used as gave complete periods. Under a and c are given the amplitudes and under ϕ_a and ϕ_c the phase constants for

TABLE V

N–S, Diurnal, Lunar

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1.....	26	0.962	0.551	185°35′	193°24′	1.222	− 7°29′
2.....	28	0.653	.470	189 59	188 19	0.973	1 40
3.....	24	1.306	.550	319 49	201 31	1.662	118 18
4.....	20	2.042	.701	305 43	218 48	2.039	86 55
5.....	25	0.117	.509	189 00	203 36	0.336	−14 36
6.....	26	.347	.465	218 59	197 7	.522	21 52
7.....	26	.335	.499	190 27	194 25	.475	− 3 58
8.....	25	.279	.484	193 22	193 25	.404	− 0 3
9.....	26	.297	.463	213 32	195 58	.451	17 34
10.....	26	.538	.473	154 44	193 31	.796	−38 37
11.....	26	.277	.455	164 41	194 4	.426	− 29 23
12.....	26	.282	.365	214 36	188 58	.541	+25 38
13.....	22	0.359	0.480	181 1	191 58	0.523	−10 57
Av_{13}...	.....	0.574	.....	.....	.....	0.777 ±0.411	11°57′ ±30 49
Av_{12}...	.....	0.478	.....	.....	.....	0.694 ±0.313	7 3 ±26 6
Av_{11} ..	.....	0.408	.....	.....	.....	.612 ±0.216	− 2 25 ±16 12
Y_{13} ...	326	0.281	0.503	229°23′	197°57′	0.411	31 26
Y_{12} ...	306	0.298	0.492	203 24	196 07	.424	7 17
Y_{11} ...	282	0.411	0.489	188 14	196 56	0.588	− 8 42

TABLE VI

E–W, Diurnal, Lunar

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1.....	26	4.107	3.819	282° 9′	284°56′	0.753	−2°47′
2.....	28	3.845	3.725	275 48	278 19	.723	−2 31
3.....	24	4.164	4.193	288 57	288 58	.695	−0 1
4.....	20	3.525	3.603	317 6	293 57	.685	23 9
5.....	25	3.925	4.134	287 31	285 43	.665	1 48
6.....	26	3.517	3.850	291 19	288 14	.639	3 5
7.....	26	3.925	3.900	281 44	285 13	.704	−3 29
8.....	25	4.032	3.968	281 54	281 24	.711	0 30
9.....	26	3.710	3.518	286 41	283 20	.738	3 21
10.....	26	3.480	3.595	282 29	284 21	.678	−1 53
11.....	26	3.383	3.442	281 40	283 59	.688	−2 19
12.....	26	3.439	3.470	278 57	281 36	.694	−2 39
13.....	22	4.526	4.485	275 17	280 35	0.706	−5 18
Av_{13}...	326	3.807	.....	.....	.....	0.699 ±0.022	0°29′ ±4 11
Av_{12} ..	306	3.825	.....	.....	.....	0.700 ±0.024	−1 0 ±2 8
Y_{13} ...	326	3.759	3.799	284°55′	283°22′	0.693	1 33
Y_{12} ...	306	3.807	3.815	283 52	282 43	0.699	1 9

the observed curves and under b and d and ϕ_b and ϕ_d the corresponding quantities for the calculated curves. In taking the means the value for each month is weighted in proportion to the number of periods in the month. Below the mean R and $\Delta\phi$ in each case is given the average difference from the mean. At the bottom of each table opposite Y are given the results obtained by computing the tides for the entire year as a single period instead of for a month at a time. The results agree closely with the means for the thirteen months, and are shown graphically in Figures 4 to 8.

Violent storms broke down the electric wires during February and March on several occasions, interrupting the electric current for a few hours. The pits cooled, and there were resulting large shifts of the fringes after the current was re-established. It seems highly probable that the large difference of phase which is shown for the fourth month E–W diurnal tide is due to such disturbances. Moreover, this month, on account of the interruptions, contained but 20 periods instead of the usual 26. The mean is therefore given for twelve lunar months, omitting the fourth, on the line Av_{12} as well as for the thirteen months. A calculation for 306 periods was also made, omitting this month from both the observed and calculated data. This value is given opposite Y_{12}.

In the case of the N–S diurnal tide it will be noted that the amplitude is very small, about 0.5 fringe, as it should be since this tide has the coefficient $\cos 2l$, where l is the latitude. For Yerkes Observatory $\cos 2l = 0.0848$. This tide is too small to admit of much accuracy in the determination, but the results are included, as they are not without interest. Mean values are added for the whole year, and the means omitting the fourth month, and also omitting both the third and the fourth. The calculations are also added for the year as a whole, 326 periods; omitting the fourth month, 306 periods; and omitting both the third and fourth months, 282 periods. The omission of these months is perhaps justified, since both R and $\Delta\phi$ are decidedly abnormal. The mean of the six values gives $R = 0.584$ and $\Delta\phi = 7°46'$. Probably the only conclusion which is justified for the N–S diurnal tide is that R is about $0.6 \pm .2$ and that the difference of phase is small.

An effort was made to deduce the fortnightly lunar tide of period 13.66 days. Here the E–W tide should be zero, and the residual sine curves were less than 0.05 fringe from both the

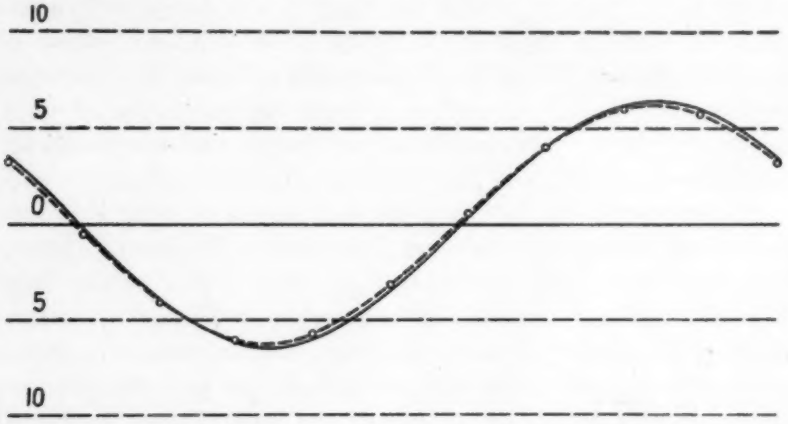


FIG. 4.—N–S semi-diurnal lunar tide for entire year. Dotted curve from observed values

$$y=6.173 \sin (\theta+148^\circ 24')$$

Full curve from 0.7 calculated values

$$y=6.401 \sin (\theta+147^\circ 32')$$
$$R=0.7\ a/b=0.675.\quad \Delta\phi=52'$$

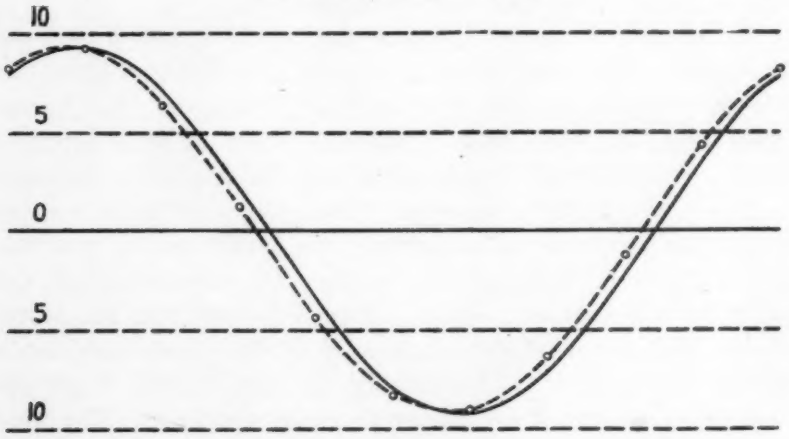


FIG. 5.—E–W semi-diurnal lunar tide for entire year. Dotted curve from observed values

$$y=9.380 \sin (\phi+63^\circ 56')$$

Full curve from 0.7 of calculated values

$$y=9.485 \sin (\theta+58^\circ 10')$$
$$R=0.7\ c/d=0.692.\quad \Delta\phi=5^\circ 46'$$

calculated and observed curves. The N–S tide, however, had an amplitude of about 1.8 fringes and gave $R=0.628$ and $\Delta\phi=-8°24'$, quantities which agree as well as could be expected with the shorter

FIG. 6.—N–S solar tide for entire year. Dotted curve from observed values

$$y=3.140 \sin(\theta+99°25')$$

Full curve from 0.7 of calculated value

$$y=3.063 \sin(\theta+93°20')$$
$$R=0.7\ a/b=0.718.\quad \Delta\phi=6°5'$$

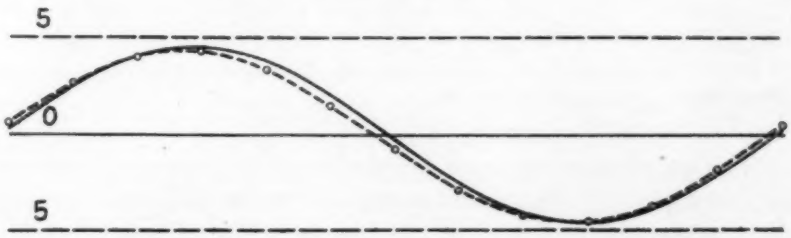


FIG. 7.—E–W solar tide for entire year. Dotted curve from observed values

$$y=4.399 \sin(\theta+8°6')$$

Full curve from 0.7 of calculated value

$$y=4.522 \sin(\theta+2°23')$$
$$R=0.7\ c/d=0.681.\quad \Delta\phi=5°43'$$

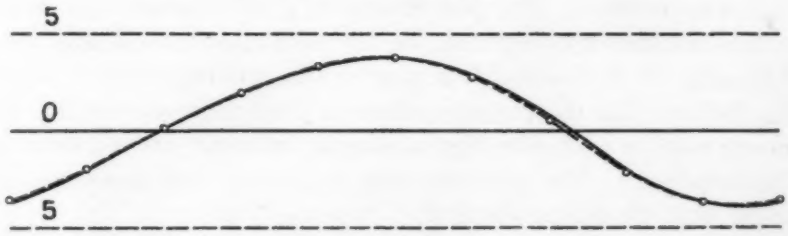


FIG. 8.—E–W diurnal lunar tide for entire year. Dotted curve from observed values

$$y=3.759 \sin(\theta+284°55')\ .$$

Full curve from 0.7 of calculated values

$$y=3.799 \sin(\theta+282°53')$$
$$R=0.7\ c/d=0.693.\quad \Delta\phi=1°33'$$

periods. The negative sign of $\Delta\phi$ merely indicates that the uncertainty is considerable.

The results are collected in Table VII. The value given in each case is the average of the mean value for the thirteen lunar months and the value deduced by treating all the observations in a single set, except in the case of the N–S diurnal tide, where the values given are the mean of the six determinations mentioned above, and in the case of the E–W diurnal tide, which is the mean of four similar determinations.

TABLE VII

	N–S			E–W		
	Amp.	R	$\Delta\phi$	Amp.	R	$\Delta\phi$
Lunar semi-diurnal	6.188	0.6765 ± .019	1°15′ ± 1°2′	9.376	0.6915 ± .007	5°54′ ± 56′
Solar	3.150	0.716 ± .041	5°53′ ± 3°12′	4.429	.681 ± .018	5°30′ ± 1°28′
Lunar diurnal	.408	0.584 ± .200	(7°46′ ± 15°)	3.799	.698 ± .023	38′ ± 3°28′
Weighted means	.....	0.6895	2°41′	.....	0.6903	4°34′
Final values		$R = 0.690 \pm .004$			$\Delta\phi = 4°$	

The amplitudes are those for the observed curves averaged as indicated above. The errors indicated for the different tides are the average differences from the means of the thirteen months, except in the case of the N–S diurnal tide, where it is simply an estimate. In combining the different tides to get a mean in each direction, the ratios and phase differences were weighted in proportion to the amplitudes of the tides except that the N–S diurnal tide was omitted. The final means of R and $\Delta\phi$ are the mean values obtained by weighting the five determinations in this way. This seems to be the most logical procedure and is perhaps justified by the fact that the average difference from the mean for the different tides in each direction is roughly inversely proportional to the amplitude. The probable error, computed in the usual way, is given with the final value of R.

The final result indicates that the rigidity of the earth in the N–S and E–W directions is the same[1] and the ratio R is 0.690 with

[1] The preliminary experiment, through an error in computation, indicated a difference in the rigidities in the two directions. The ratio should have been 0.710 for both the N–S and E–W. See *Science*, October 3, 1919, p. 327.

a probable error of ± 0.004. That the viscous yielding of the earth is small is indicated by the small difference in phase between the observed and computed tides. It will be noted that the two solar tides appear to agree excellently in phase displacement with the E–W semi-diurnal tide and that for the N–S semi-diurnal, and probably also the E–W diurnal lunar tides the phase displacement is definitely smaller.

However, for lack of a better method of finding the means of the N–S and E–W phase displacement, each was averaged as the ratios were, that is, simply by weighting them in proportion to the amplitudes of the tides. This gives a displacement in phase of the water tides in the N–S direction of $+2°41'$ and in the E–W direction $+4°34'$. Although it seems certain that the difference in phase is slightly larger in the E–W than in the N–S direction, a mean displacement of $+4°.0$ is probably correct to within $1°$. If we take $R = 0.690$, the tides in the actual earth are 0.310 of what they would be if the earth were fluid, and the value of $\Delta\phi$ equal to $4°.0$, for the displacement of the water tides means that the earth tides lag behind the impressed forces by this same amount.

It is desired to express appreciation of the interest taken in this work by Professor T. C. Chamberlin, Professor E. B. Frost, and Professor F. R. Moulton.

RYERSON LABORATORY
November 1919

THEORY OF TIDES IN PIPES ON A RIGID EARTH

By F. R. MOULTON

The moon and sun exert tidal forces upon the earth. The magnitudes of these forces depend upon the masses and distances of these bodies and upon the size of the earth, all of which are known. If the earth were perfectly rigid, the tides in surface fluids would have certain definite magnitudes. Since the earth as a whole yields somewhat to the tidal forces, the actual tides in surface fluids are less than they would be if the earth were perfectly rigid. The amount the earth is deformed and the character of its yielding can be determined from the differences in magnitudes and phases between the actual tides and what they would be on a rigid earth. The former are found from observations, and the latter can be determined only by computations. The theory of making these computations is the object of this paper.

Lord Kelvin inferred from the height of the oceanic tides that the earth as a whole has a considerable degree of rigidity. But oceanic tides are not suited to a precise numerical determination of the earth's rigidity because they are modified by many factors whose effects cannot be even approximately evaluated. Hence it is necessary to use relatively short channels of water, or their equivalent. This reduces the magnitudes of the quantities to be measured. Nevertheless, in the experiments of Michelson and Gale, in which the pipes are only about 500 feet long, the results are so precise that, in order that the theory shall be as accurate as the observations, it is necessary to develop the tidal theory to an order of precision not heretofore required.

The surface of the water in a partially filled pipe is an equipotential surface for all the forces which act upon it. The periods of any waves which might be produced in it would be so short that they could not be confused with tidal oscillations, and, moreover, they would soon be damped out. Hence it is necessary only to develop the expressions for the equipotential surfaces, and to find the way they vary with the time.

Take the origin O at the center of the earth and let

P_0 and P_1 be the positions of the ends of the pipe,
r be the distance from O to P_0,
ϕ be the latitude of P_0,
θ be the right ascension of P_0,
ω be the angular rate of rotation of the earth,
E be the mass of the earth,
a be the earth's equatorial semidiameter,
b be the earth's polar semidiameter,
e be the eccentricity of a meridian section of the earth,
M be the mass of the moon,
α be the right ascension of the moon,
δ be the declination of the moon,
ρ be the distance from O to the moon,
ρ_1 be the distance from P_0 to the moon,
ψ be the angle between r and ρ,
S be the mass of the sun,
A be the right ascension of the sun,
D be the declination of the sun,
R be the distance from O to the sun,
R_1 be the distance from P_0 to the sun,
Ψ be the angle between r and R,
k^2 be the gravitational constant,
V_0 be the potential function at P_0,
V be the potential function at the general point P.

The potential function is composed of four parts: V_1, that due to the attraction of the earth; V_2, that due to the rotation of the earth; V_3, that due to the attraction of the moon; and V_4, that due to the attraction of the sun. The first of these is numerically much more important than the remainder, and this fact will be used in certain expansions in series which follow. The earth is somewhat oblate, and this fact must be taken into account in order that the theory shall be at least as accurate as the observations of Michelson and Gale.

The first part of the potential function is[1]

$$V_1=\frac{k^2E}{r}\left\{1+1\tfrac{1}{10}\frac{b^2}{r^2}(\cos^2\phi-2\sin^2\phi)e^2\ \ldots\ .\right\}. \qquad (1)$$

[1] *Introduction to Celestial Mechanics*, p. 122.

It follows from the formulae for centrifugal acceleration that the second part is

$$V_2=\tfrac{1}{2}\omega^2 r^2\cos^2\phi. \tag{2}$$

The third and fourth parts are[1] the similar functions

$$V_3=k^2M\left\{\frac{1}{\rho_1}-\frac{r}{\rho^2}\cos\psi\right\}, \tag{3}$$

$$V_4=k^2S\left\{\frac{1}{R_1}-\frac{r}{R^2}\cos\Psi\right\}, \tag{4}$$

and

$$V=V_1+V_2+V_3+V_4. \tag{5}$$

The condition that P_0 and P shall be on the same equipotential surface is

$$V-V_0=0. \tag{6}$$

The polar co-ordinates of V_0 are r, ϕ, θ; let the polar co-ordinates of V be represented by $r+\Delta r$, $\phi+\Delta\phi$, $\theta+\phi\theta$, where, in the case of pipes such as can be used in experiments, Δr, $\Delta\phi$, and $\Delta\theta$ are small quantities. Then the expansion of (6) as a power series in Δr, $\Delta\phi$, and $\Delta\theta$ is

$$\left.\begin{aligned}0=\frac{\delta V}{\delta r}\Delta r+\frac{\delta V}{\delta\phi}\Delta\phi+\frac{\delta V}{\delta\theta}\Delta\theta+\tfrac{1}{2}\frac{\delta^2 V}{\delta r^2}(\Delta r)^2+\tfrac{1}{2}\frac{\delta^2 V}{\delta\phi^2}(\Delta\phi)^2+\tfrac{1}{2}\frac{\delta^2 V}{\delta\theta^2}(\Delta\theta)^2\\ +\frac{\delta^2 V}{\delta r\delta\phi}\Delta r\Delta\phi+\frac{\delta^2 V}{\delta r\delta\theta}\Delta r\Delta\theta+\frac{\delta^2 V}{\delta\phi\delta\theta}\Delta\phi\Delta\theta+\ \ldots\ldots\end{aligned}\right\} \tag{7}$$

The problem is to find Δr from this equation when $\Delta\phi$ and $\Delta\theta$ are known. Equation (7) has the form

$$\left.\begin{aligned}0=c\Delta r+a_1\Delta\phi+a_2\Delta\theta+a_3(\Delta r)^2+a_4(\Delta\phi)^2+a_5(\Delta\theta)^2+a_6\Delta r\Delta\phi+a_7\Delta r\Delta\theta\\ +a_8\Delta\phi\Delta\theta+\ \ldots\ldots\end{aligned}\right\} \tag{8}$$

which can be solved for Δr as a power series in $\Delta\phi$ and $\Delta\theta$ provided c is not zero, a condition which is fulfilled in this problem, because $c=0$ would imply that gravity is zero at P_0. The solution of (8) has the form

$$\Delta r=c_1\Delta\phi+c_2\Delta\theta+c_3(\Delta\phi)^2+c_4(\Delta\theta)^2+c_5\Delta\phi\Delta\theta+\ \ldots\ldots \tag{9}$$

[1] *Op. cit.*, p. 372.

On substituting (9) in (8) and equating coefficients of corresponding powers of $\Delta\phi$ and $\Delta\theta$, it is found that

$$\left.\begin{aligned} c_1 &= -\frac{a_1}{c}, \\ c_2 &= -\frac{a_2}{c}, \\ c_3 &= -\frac{a_4}{c}+\frac{a_1a_6}{c^2}-\frac{a_1^2a_3}{c^3}, \\ c_4 &= -\frac{a_5}{c}+\frac{a_2a_7}{c^2}-\frac{a_2^2a_3}{c^3}, \\ c_5 &= -\frac{a_8}{c}+\frac{a_1a_7}{c^2}+\frac{a_2a_6}{c^2}-\frac{2a_1a_2a_3}{c^3}. \end{aligned}\right\} \tag{10}$$

The higher terms are all negligible, and a number of those written will be shown to be numerically unimportant. In computing the second derivative of V with respect to r it will be sufficient to retain only the principal part of V_1.

It follows from the definitions of r, ρ, ψ, ρ_1, R, R_1, and Ψ that

$$\left.\begin{aligned} \rho_1^2 &= r^2+\rho^2-2r\rho\cos\psi, \\ R_1^2 &= r^2+R^2-2rR\cos\Psi. \end{aligned}\right\} \tag{11}$$

Then it follows from equations (1), . . . , (11) that

$$\left.\begin{aligned} c=\frac{\delta V}{\delta r} &= -\frac{k^2E}{r^2}\left\{1+\tfrac{3}{10}\frac{b^2}{r^2}(\cos^2\phi-2\sin^2\phi)e^2+\ \ldots\ .\right\}+\omega^2r\cos^2\phi \\ &\quad -k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta r}+\frac{1}{\rho^2}\cos\psi\right\}-k^2S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta r}+\frac{1}{R^2}\cos\Psi\right\}, \\ a_1=\frac{\delta V}{\delta\phi} &= -\frac{k^2E}{r^2}\frac{\delta r}{\delta\phi}-\tfrac{3}{10}k^2E\,\frac{b^2e^2}{r^3}\sin 2\phi-\tfrac{1}{2}\omega^2r^2\sin 2\phi \\ &\quad -k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\phi}-k^2S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta\cos\Psi}+\frac{r}{R^2}\right\}\frac{\delta\cos\Psi}{\delta\phi}, \\ a_2=\frac{\delta V}{\delta\theta} &= -k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\theta} \\ &\quad -k^2S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta\cos\Psi}+\frac{r}{R^2}\right\}\frac{\delta\cos\Psi}{\delta\theta}, \\ a_3=\tfrac{1}{2}\frac{\delta^2V}{\delta r^2} &= \frac{k^2E}{r^3}, \qquad a_4=\tfrac{1}{2}\frac{\delta^2V}{\delta\phi^2}=\tfrac{1}{2}\frac{\delta a_1}{\delta\phi}, \qquad a_5=\tfrac{1}{2}\frac{\delta^2V}{\delta\theta^2}=\tfrac{1}{2}\frac{\delta a_2}{\delta\theta}, \\ a_6=\frac{\delta^2V}{\delta r\delta\phi} &= \frac{\delta a_1}{\delta r}, \qquad a_7=\frac{\delta^2V}{\delta r\delta\theta}=\frac{\delta a_2}{\delta r}, \qquad a_8=\frac{\delta^2V}{\delta\phi\delta\theta}=\frac{\delta a_1}{\delta\theta}. \end{aligned}\right\} \tag{12}$$

The east-and-west tide is obtained by putting $\Delta\phi = 0$, and the north-and-south tide by putting $\Delta\phi = 0$. Since in the tide experiment pipes are used only in these two directions, it is not necessary to compute c_5 and a_8. The parts of the other a_i which may be neglected can be determined only by deciding what accuracy must be attained and by passing to numbers. The degree of accuracy to be attained in the calculations depends upon that attained in the observations. In the Michelson-Gale experiments the individual observations are accurate to within about 1 per cent, and their averages, of course, are somewhat more exact. Consequently, it will be sufficient to determine the theoretical tides to within one-tenth of 1 per cent of their values. All terms will be included which affect the results by as much as this amount. In order to determine what terms must be included it is necessary to adopt numerical values for the various quantities involved.

The units of distance, time, and mass will be taken respectively as the mean distance from the earth to the sun, the mean solar day, and the mass of the sun. In these units

$$\left.\begin{aligned} E &= \frac{1}{330{,}000}, \\ \frac{M}{E} &= \frac{1}{81.8}, \\ \omega &= \frac{\text{sidereal day}}{\text{solar day}} 2\pi, \\ \log k &= 8.23558 - 10, \\ a &= \frac{3963.3}{92{,}900{,}000}, \\ b &= \frac{3950}{92{,}900{,}000}, \\ \text{mean } \rho &= \frac{238{,}860}{92{,}900{,}000}, \\ e^2 &= 0.00665, \\ \phi \text{ of } P &= 42^\circ 34' 13''. \end{aligned}\right\} \tag{13}$$

Since

$$r = \frac{b}{[1 - e^2 \cos^2 \phi]^{\frac{1}{2}}}, \tag{14}$$

it follows that

$$\left.\begin{aligned} r&=\frac{3950}{92{,}900{,}000}[1+\tfrac{1}{2}e^2\cos^2\phi\ \ldots\ .] \\ &=\frac{3957}{92{,}900{,}000}\text{ at }\phi=42^\circ 34'13''. \end{aligned}\right\} \tag{15}$$

Now consider the expression for c, equations (12). Let $-k^2E/r^2$ be taken out as a factor. Then the smaller quantities will be compared with unity. It follows from (14) that

$$\left.\begin{aligned} \tfrac{3}{10}\frac{b^2}{r^2}(\cos^2\phi-2\sin^2\phi)e^2&=-\tfrac{3}{10}[2-(3+2e^2)\cos^2\phi+3e^2\cos^4\phi]e^2 \\ &=-0.00074\text{ for }\phi=42^\circ 34'13''. \end{aligned}\right\} \tag{16}$$

It is also found that

$$\left.\begin{aligned} \frac{-\omega^2r^3\cos^2\phi}{k^2E}&=-0.00338(1+\tfrac{3}{2}e^2\cos^2\phi)\cos^2\phi=-0.00184\text{ for} \\ &\qquad\qquad \phi=42^\circ 34'13''. \end{aligned}\right\} \tag{17}$$

These quantities are constants for a given station.

Now consider the third term of c. It follows from (11) that

$$\left.\left|\frac{\delta\rho_1}{\delta r}\right|=\left|\frac{-\rho\cos\psi+r}{\rho_1}\right|\leqq 1.\right\} \tag{18}$$

Hence, at the maximum,

$$\frac{k^2M}{k^2E}\left|\frac{r^2}{\rho_1^2}\frac{\delta\rho_1}{\delta r}-\frac{r^2}{\rho^2}\cos\psi\right|\leqq 0.000006.$$

This term is therefore quite insensible, and the same is true of the last one. Hence,

$$\left.\begin{aligned} \frac{\delta V}{\delta r}=c&=-\frac{k^2E}{r^2}[0.99601+0.00261\cos^2\phi] \\ &=-1.0027\frac{k^2E}{a^2}[1-0.00406\cos^2\phi] \\ &=-1.0005\frac{k^2E}{a^2}\text{for }\phi=42^\circ 34'13''. \end{aligned}\right\} \tag{19}$$

Now consider the expression for a_1. The first three terms are constant for a given station, and, since only the variations in the tides can be measured, they may be omitted.

Now consider the fourth expression in a_1. It follows from (18) that

$$\frac{\delta\rho_1}{\delta\cos\psi}=-\frac{\rho}{\rho_1}r. \tag{20}$$

It follows from the definition of the symbols ψ, ϕ, δ, α, θ, Ψ, D, and A that

$$\left.\begin{aligned}
\cos\psi&=\cos\delta\cos\phi\cos(\alpha-\theta)+\sin\phi\sin\delta,\\
\cos\Psi&=\cos D\cos\phi\cos(A-\theta)+\sin\phi\sin D,\\
\frac{\delta\cos\psi}{\delta\phi}&=-\cos\delta\sin\phi\cos(\alpha-\theta)+\cos\phi\sin\delta,\\
\frac{\delta\cos\psi}{\delta\theta}&=\cos\delta\cos\phi\sin(\alpha-\theta),\\
\frac{\delta\cos\Psi}{\delta\phi}&=-\cos D\sin\phi\cos(A-\theta)+\cos\phi\sin D,\\
\frac{\delta\cos\Psi}{\delta\theta}&=\cos D\cos\phi\sin(A-\theta).
\end{aligned}\right\} \tag{21}$$

Therefore the fourth term in a_1 becomes

$$-k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\phi}=-k^2Mr\rho\left[\frac{1}{\rho^3}-\frac{1}{\rho_1^3}\right]\frac{\delta\cos\psi}{\delta\phi}. \tag{22}$$

Since ρ and ρ_1 are nearly equal it will be most convenient to expand the difference $\frac{1}{\rho^3}-\frac{1}{\rho_1^3}$ into a series. It follows from (11) that

$$\left.\begin{aligned}
\frac{1}{\rho^3}-\frac{1}{\rho_1^3}&=\frac{1}{\rho^3}\left\{1-\left[1-\frac{2r}{\rho}\cos\psi+\left(\frac{r}{\rho}\right)^2\right]^{-\frac{3}{2}}\right\}=\frac{r}{\rho^4}\left\{3\cos\psi\right.\\
&\left.+\tfrac{3}{4}\frac{r}{\rho}(3+5\cos 2\psi)+\tfrac{5}{8}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos 3\psi)\ \ldots\ldots\right\}.
\end{aligned}\right\} \tag{23}$$

The maximum numerical values of the terms included in the bracket are

$$3,\quad 6\frac{r}{\rho},\quad 10\left(\frac{r}{\rho}\right)^2,\quad 15\left(\frac{r}{\rho}\right)^3,\ \ldots\ldots$$

Since r/ρ is about 1/60, it is sufficient, in order to secure an accuracy of one-tenth of one per cent, to retain the first three terms in the right member of (23). The last two terms of a_1 are therefore

$$\left.\begin{aligned} -k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\phi} &= -3k^2M\frac{r^2}{\rho^3}\left\{\cos\psi+\tfrac{1}{4}\frac{r}{\rho}(3+5\cos 2\psi)\right. \\ &\quad \left.+\tfrac{5}{24}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos 3\psi)\ \ldots\ .\right\}\frac{\delta\cos\psi}{\delta\phi}, \\ -k^2S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta\cos\Psi}+\frac{r}{R^2}\right\}\frac{\delta\cos\Psi}{\delta\phi} &= -3k^2S\frac{r^2}{R^3}\left\{\cos\Psi+\ \ldots\ .\right\}\frac{\delta\cos\Psi}{\delta\phi}, \end{aligned}\right\} \quad (24)$$

one term being sufficient in the right member of the second equation because r/R is about $1/23{,}000$.

The north-and-south tides of the first order in $\Delta\phi$ are therefore given by

$$(\Delta r)_{NS}=-\frac{a_1}{c}\Delta\phi=(Af+BF)r\Delta\phi, \quad (25)$$

where

$$\left.\begin{aligned} A &= -0.036454(1+0.0074\cos^2\phi)\left(\frac{a}{\rho}\right)^3\left\{\cos\psi+\tfrac{1}{4}\frac{r}{\rho}(3+5\cos 2\psi)\right. \\ &\quad \left.+\tfrac{5}{24}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos 3\psi)+\ \ldots\ .\right\} \\ B &= -984030(1+0.0074\cos^2\phi)\left(\frac{a}{R}\right)^3\left\{\cos\Psi+\ \ldots\ .\right\} \\ f &= -\cos\delta\sin\phi\cos(\alpha-\theta)+\cos\phi\sin\delta, \\ F &= -\cos D\sin\phi\cos(A-\theta)+\cos\phi\sin D. \end{aligned}\right\} \quad (26)$$

The east-and-west tides of the first order in $\Delta\theta$ are similarly given by

$$(\Delta r)_{EW}=-\frac{a_2}{c}\Delta\theta=(Ag+BG)\cos\phi\ r\Delta\phi, \quad (27)$$

where

$$\left.\begin{aligned} g &= \cos\delta\sin(\alpha-\theta), \\ G &= \cos D\sin(A-\theta). \end{aligned}\right\} \quad (28)$$

It will now be shown that the terms of higher order in (9) are insensible for the present problem. Consider the third term, whose coefficient is given in (10), in comparison with the first term. The ratio is found from (10) to be

$$\frac{c_3}{c_1}\Delta\phi=\left(\frac{a_4}{a_1}-\frac{a_6}{c}+\frac{a_1a_3}{c^2}\right)\Delta\phi. \quad (29)$$

It follows from (12) and (25) that the principal part of a_4/a_1 is

$$\frac{\frac{\delta}{\delta\phi}(\cos\psi \cdot f)}{\frac{1}{2}\cos\psi \cdot f},$$

which is of the order of unity. It follows from (11), (12), (22), and (24) that the principal part of $-\frac{a_6}{c}$ is

$$-6\frac{M}{E}\left(\frac{r}{\rho}\right)^3 \cos\psi \frac{\delta \cos\psi}{\delta\phi},$$

which is very small compared to unity. Finally, the principal part of $\frac{a_1 a_3}{c^2}$ is

$$-3\frac{M}{E}\left(\frac{r}{\rho}\right)^3 \cos\psi,$$

which also is very small compared to unity. Since $\Delta\phi$ is less than 1/40,000 for a pipe 500 feet long, the term in question is wholly inappreciable. The same is true of the fourth term of (9), and the last vanishes for both north-and-south and east-and-west pipes.

The theoretical tides in the pipes on a rigid earth are given by equations (25), . . ., (28). The quantities $r\Delta\phi$ and $\cos\phi\, r\Delta\theta$ are the lengths of the north-and-south and east-and-west pipes respectively, and are constant in the computations for a given station. They may be expressed in any units which may be convenient in comparing the computed and observed results. Let

$$\left.\begin{aligned} k_1 &= -0.036454\, r\Delta\phi, \\ k_2 &= -0.036454 \cos\phi\, r\Delta\theta, \\ K_1 &= -984030\, r\Delta\phi, \\ K_2 &= -984030 \cos\phi\, r\Delta\theta \end{aligned}\right\} \tag{30}$$

If the two pipes are equal in length, then $k_1 = k_2$, $K_1 = K_2$.

The latitude ϕ is constant for each station. The quantities α, δ, ρ, A, D, and R are taken from the *Ephemeris*. The sidereal time depends upon the times of observation. If they differ by equal intervals, say two hours of mean solar time, the values of θ differ by about $30°5'$, or, more exactly, $30°4'55''.7$. The quantities $\cos\psi$ and $\cos\Psi$ are computed from the first two equations of (21). Then equations (25), . . ., (28) give the desired results.

The terms

$$Q=\tfrac{1}{4}\frac{r}{\rho}(3+5\cos 2\psi)+\tfrac{5}{24}\left(\frac{r}{\rho}\right)^{2}(9\cos\psi+7\cos 3\psi) \qquad (31)$$

of equations (25) and (27) are small corrections to $\cos\psi$. Since they are relatively small, a table for Q with argument $\cos\psi$ was computed, which is given herewith. The use of this table made it unnecessary to compute Q for each value of $\cos\psi$ encountered in computing the tides.

TABLE I

Cos ψ	Q	Cos ψ	Q	Cos ψ	Q
1.00	0.0343	0.30	−0.0047	−0.40	−0.0015
0.95	.0300	.25	− .0059	− .45	− .0003
.90	.0260	.20	− .0068	− .50	− .0022
.85	.0222	.15	− .0075	− .55	− .0044
.80	.0186	.10	− .0080	− .60	− .0067
.75	.0153	.05	− .0083	− .65	− .0093
.70	.0122	00	− .0083	− .70	− .0120
.65	.0093	− .05	− .0082	− .75	− .0149
.60	.0066	− .10	− .0078	− .80	− .0181
.55	.0042	− .15	− .0073	− .85	− .0214
.50	.0019	− .20	− .0065	− .90	− .0249
.45	− .0001	− .25	− .0056	− .95	− .0285
.40	− .0018	− .30	− .0044	−1.00	− .0324
.35	− .0034	− .35	− .0031		

The variations in r and ρ produce no sensible variations in the last term of Q. The variations in ρ, however, in the first term of Q have small effects which must be taken into account in order to secure an accuracy of one part in a thousand. The ratio r/ρ is defined in the *Ephemeris* by the parallax of the moon. The following table gives the correction δQ to Q to be applied as a consequence of the variations in the moon's parallax.

TABLE II

Moon's Parallax	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ
	1.00	0.80	0.50	0.25	.00	−0.25	−0.50	−0.80	−1.00
60′	0.0017	0.0009	0.0001	−0.0003	−0.0004	−0.0003	0.0001	0.0009	0.0016
58′.5	.0008	.0005	.0000	− .0002	− .0002	− .0001	.0000	.0005	.0008
57′	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
55′.5	−.0008	−.0005	.0000	.0002	.0002	.0001	.0000	−.0005	−.0008
54′	−.0017	−.0009	−.0001	.0003	.0004	.0003	−.0001	−.0009	−.0016

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ON THE BRIGHTNESS OF THE SKY AT NIGHT AND THE TOTAL AMOUNT OF STARLIGHT[1]

By P. J. VAN RHIJN

1. INTRODUCTION

The following is an abstract of the results of an investigation which will appear in the "Papers from the Mount Wilson Observatory." It contains a discussion of observations of the brightness of the sky made at Mount Wilson during the summer and autumn of 1913. The main points investigated are the total amount of starlight per square degree for each galactic latitude and the distribution over the sphere of the illumination of the sky which is not due to the stars. The latter appears to vary with the latitude and longitude of the area observed; the evidence indicates that at least part of the light of the sky proceeds from a source probably identical with that of the zodiacal light.

2. THE OBSERVATIONS

The instrument used is that described by Yntema.[2] For details as to the method of measurement, reference may be made to the extended account of the investigation. Here it need only be remarked that the determination of the brightness of the sky has been made in two steps: (*a*) comparison of the light of various regions of the sky with that of an area at the North Pole; (*b*) comparison of the light at the pole with that of a star of known magnitude.

The first step has been made by varying the illumination of an annular screen in a measurable way until it disappeared against the background of the sky. The amount of light per square degree at the pole has been determined in terms of starlight by first comparing the surface brightness of the sky near the pole with that of an artificially illuminated disk of opal glass, and then estimating the stellar magnitude of the disk by observing it from a distance such that it had the appearance of a star.

[1] *Contributions from the Mount Wilson Observatory*, No. 173.

[2] *Groningen Publications*, No. 22, pp. 9–11, 1909.

The small electric lamps used in the photometers were fed by a two-cell storage battery. The constancy of the current was carefully controlled. The only photometric principle used in the measurements is the law of inverse squares.

The areas measured were identified by means of a map showing all the stars visible to the naked eye. The time of observation was noted in all cases.

The observations made in series are as follows: (1) measures of the brightness at the North Pole, which was determined several times each night; (2) measures of areas on a circle parallel to the horizon; (3) measures on a circle perpendicular to the horizon.

3. GENERAL PLAN OF THE INVESTIGATION

The observations have been used to investigate the following questions: (*a*) Yntema found that the brightness of the sky is not due exclusively to the stars,[1] a result confirmed by the present data. What is the distribution of this additional light over the sphere and what is its cause? (*b*) Is it possible to eliminate the additional light and thus discuss the distribution of starlight alone?

The first of these questions may be answered by investigating the skylight for galactic latitudes higher than 40°. Above this limit the light due to the stars is a relatively small percentage of the total brightness and can be computed with sufficient accuracy from the counts of stars given in *Groningen Publications*, No. 27, Table V. We thus find for all stars fainter than magnitude 5.5 the results shown in Table I. These may be considered as the quantities of starlight actually measured with our photometer.

TABLE I

LIGHT PER SQUARE DEGREE OF ALL STARS FAINTER THAN MAG. 5.5

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Gal. Lat.	Amount of Light
40°	0.012
50	.011
60	.009
70	.009
80	.008
90	0.008

[1] L. Yntema, *op. cit.*, p. 35.

Yntema found that the so-called earthlight, i.e., the brightness not due to the stars,[1] changes with the zenith distance of the area observed, but that it is approximately independent of the azimuth. I first investigated, by means of measures on a circle parallel to the horizon, whether the earthlight is really the same for all points on such a circle. Only those areas were used whose galactic latitudes exceed 40°. The earthlight was found by subtracting the direct and scattered starlight from the total brightness. The differences vary clearly with the azimuth, being always larger in the eastern part of the sky. It will be shown that these variations are caused by a kind of zodiacal light extending over the whole sky. The observed brightness corrected for starlight depends distinctly on the latitude and longitude relative to the sun. The excess of this zodiacal brightness over its mean value will be found for each latitude and longitude,[2] and a correction equal to this excess with opposite sign has been applied to all measures. The values of the earthlight thus corrected are independent of the azimuth; in any series of measures parallel to the horizon we can thus find the starlight for areas in the Milky Way by taking the difference between the total observed brightness and the corrected[3] value of the earthlight, the amount of which has been found from areas in higher galactic latitudes. This gives the solution of the second problem stated above.

I next considered the dependence of the corrected earthlight on the zenith distance by means of the series of measures in high galactic latitudes along circles perpendicular to the horizon. The values increase with increasing zenith distance. This is partly due to earthlight scattered by the atmosphere, the scattering being greatest near the horizon. The amount of scattered earthlight was found by means of Abbot's determination of the ratio of direct and scattered sunlight.[4] After subtraction of the scattered earthlight, the residuals, corrected for the absorption of the atmosphere,

[1] The starlight scattered by the atmosphere is *not* contained in the earthlight. The determination of its amount will be described below.

[2] Longitude means always longitude relative to the sun.

[3] That is, corrected for the excess of zodiacal light over its mean value.

[4] *Astronomical Journal*, **28**, 130, 1914.

still increase toward the horizon. An empirical formula which represents these residuals was found and used to determine the earthlight for areas near the Galaxy. Subtraction of the values thus found from the observed brightness then gave the starlight in the lower galactic latitudes. This is another solution of the second question.

From the preceding lines it appears that I have based my conclusions as much as possible on a comparison of measures included in the *same* series of observations, which were always finished within one hour. This has been done in order to eliminate any variation of the earthlight with the time.[1]

4. CHANGE OF THE SKY-BRIGHTNESS NEAR THE NORTH POLE

The brightness at the North Pole has been measured repeatedly every night. Its value appears to be subject to large variations. Even during the same night the light near the pole may change as much as 20 per cent. These fluctuations are due to a variation of the earthlight. I have not, however, been able to find any systematic tendency in these variations, although the mean brightness is practically the same before and after midnight. The changes from one night to another are rather capricious.

5. DEPENDENCE OF THE EARTHLIGHT ON THE AZIMUTH. ZODIACAL LIGHT EXTENDING OVER THE WHOLE SKY

All series of observations parallel to the horizon have been treated in the way indicated in Table II, which serves only as an example.

The first six columns of Table II give the number, the galactic latitude (b), the latitude (β), the longitude relative to the sun (λ), the zenith distance (z), and the azimuth (A). The co-ordinates have been computed from the right ascension, the declination, and the time of observation. B denotes the observed amount of light per square degree expressed in terms of that of a star of magnitude 1.00 (Harvard Scale). The earthlight for areas whose galactic latitudes exceed 40° is given under the heading $B_{s'-s}$. This has been formed by subtracting the direct starlight (s) and the scattered

[1] See section 4.

TABLE II

REDUCTION OF A SERIES OF OBSERVATIONS PARALLEL TO THE HORIZON

(Unit = a star of mag. 1.00, Harvard Visual Scale)

No. (1)	b (2)	β (3)	λ (4)	z (5)	A (6)	B (7)	$B_{s'}$ (8)	$B_{s',s}$ for $b>40°$ Uncorr. (9)	$B_{s',s}$ for $b>40°$ Corr.* (10)	$B_{s'}$ Corr.* $b<40°$ (11)	Corr. Zod. L. (12)	$B^z_{s',s}$ $b>40°$ (13)	$B^z_{s'}$ $b<40°$ (14)	Star-light $b<40°$ (15)
240....	North Pole	.....	.....	.....	.....	0.118	.....	.....	.....	.....	.....	.....	.....	.....
241....	+56°	+47°	+ 82°	56°	108°	.108	0.101	0.092	0.103	.....	+0.004	0.107	.....	.....
242....	+53	+65	+ 54	53	137	.104	.097	.088	.097	.....	+ .016	.113	.....	.....
243....	+49	+69	+ 42	53	144	.100	.093	.083	.091	.....	+ .019	.110	.....	.....
244....	− 1	+47	− 84	57	214	.156	.149	.....	.....	0.168	+ .004	.....	0.172	+0.061
245....	+ 5	+54	− 79	55	207	.132	.125	.....	.....	.140	+ .010	.....	.150	+ .039
246....	+11	+58	− 72	54	199	.118	.111	.....	.....	.123	+ .012	.....	.135	+ .024
247....	−12	+40	− 93	57	228	.140	.133	.....	.....	.150	+ .002	.....	.152	+ .041
248....	−55	+ 5	−136	57	291	.122	.115	.106	.120	.....	− .010	.110	.....	.....
249....	−61	− 2	−145	58	307	.128	.120	.112	.128	.....	− .011	.117	.....	.....
250....	+25	+29	+122	55	72	.130	.123	.....	.....	.138	+ .001	.....	.139	+ .028
5[21]....	North Pole	.....	.....	.....	.....	.119	.....	.....	.....	.....	.....	.....	.....	.....
Mean..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.111	.....	.....

* Corrected for atmospheric absorption, reduced to the zenith.

starlight (s') from the measured brightness B. As has been explained already, the direct starlight per square degree has been derived from counts of stars (see Table I). The scattered starlight has been computed by means of Abbot's observations already quoted. For the same zenith distance the earthlight, in nearly all cases, is larger for areas near azimuth 270° than for those near azimuth 110°. This is the case in the series of Table II. This difference is to be explained by a dependence of the earthlight on the position of the measured area relative to the ecliptic.

That this is really the case is shown in Table III, which contains the differences between the earthlight for the smallest and the largest latitudes occurring in each series. Excepting one case, for which the difference in β is only 12°, the lower latitude corresponds always to a higher illumination.

TABLE III

DIFFERENCES IN EARTHLIGHT, SMALLEST AND LARGEST LATITUDES IN EACH SERIES PARALLEL TO THE HORIZON

β_1	β_2	Diff.	β_1	β_2	Diff.
22°	64°	+0.019	0°	70°	+0.032
21	69	+ .022	11	59	+ .041
27	69	+ .033	2	69	+ .037
58	70	− .005	9	67	+ .043
14	73	+ .025	2	23	+ .009
1	63	+0.037	7	19	+ .006
			10	63	+0.031

If now the brightness of the sky depends on the latitude, it is probably connected with the illumination usually called the zodiacal light, and must then also vary with the longitude relative to the sun. We have therefore to find the dependence of the earthlight on the latitude and longitude. It is impossible, however, to derive directly the absolute amount of the light which is related to the ecliptic and which for convenience will be called zodiacal light. For, as will be shown presently, it is very probable that the brightness of the sky is produced in part by other causes, whose contribution to the general illumination also varies with the zenith distance in a manner which is unknown. We therefore derive, first of all,

not the absolute amount of zodiacal light, but the excess of zodiacal light over its mean value for each latitude and longitude. This has been done on the supposition that the zodiacal light is the same north and south of the ecliptic; the sign of the longitude relative to the sun has also been disregarded.[1] The difference in the intensity of zodiacal light for various latitudes and longitudes can be found by subtracting any two values of the earthlight $B_{s',s}$ in the same series parallel to the horizon, for it can be shown that the variation of the earthlight with the azimuth is due to the zodiacal light alone. It follows, for instance, from the comparison of the mean $B_{s',s}$ for observations Nos. 241 to 243 with that for Nos. 248 and 249 that the zodiacal glow at $\beta=3°$, $\lambda=140°$, is $0.124-0.097=+0.027$ larger than at $\beta=60°$, $\lambda=59°$. By combining the differences thus found it is possible to compute for any latitude and longitude the excess of zodiacal light over a certain mean value.

The method used for the combination of the various differences has been described at length in the detailed investigation. It need only be stated here that the values of the excess of zodiacal light over its mean value were distributed among four groups having latitudes near $\beta=0°$, 20°, 55°, and 70°, respectively. A small correction was applied in order to reduce the figures to the exact values of these latitudes. The corrected light intensities were then plotted as a function of the longitude. Finally, in order to find the absolute amount of zodiacal light, the values of the excess were increased by +0.069. The quantity +0.069, to be derived in a later section, is somewhat hypothetical and the absolute values are less trustworthy than their differences for any two values of the latitude and longitude.

The graphically interpolated values of the zodiacal light thus found are given in Table IV. It may be remarked that Fessenhoff's[2] observations have been used for the smaller longitudes near the ecliptic and that in my measures special attention has been given to the brightness of the counterglow at $\beta=0°$, $\lambda=180°$.

[1] The longitude relative to the sun has been counted from 0° to +180° and 0° to −180°; the longitude of the sun itself is, of course, zero.

[2] *La Lumière Zodiacale.* Thèse de doctorat, 1919.

We have now to derive an interpolation formula for the zodiacal light as a function of the latitude and the longitude. This formula will be used mainly to compute the excess of zodiacal light at any

TABLE IV

VALUES OF THE ZODIACAL LIGHT

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Lat.	Long. from Sun	Zodiacal Light	Wt.	O−C	Lat.	Long. from Sun	Zodiacal Light	Wt.	O−C
0°	40°	0.329	1⁵	+0.007	20°	40°	0.183	1⁵	−0.016
0	60	.184	1⁵	− .001	20	70	.125	1⁵	+ .012
0	70	.149	2	− .001	20	100	.081	7	.000
0	80	.127	5	.000	20	130	.069	9	− .002
0	100	.098	12	+ .001	20	160	.072	12	− .001
0	130	.084	11	+ .005	55	40	.069	5	+ .007
0	160	.083	13	+ .001	55	100	.058	10	+ .004
0	170	.083	15	− .005	70	40	.047	7	− .004
0	180	0.097	4	+0.001	70	100	0.047	3	−0.002

point (β, λ) over the mean value. After the application of this excess with the reversed sign to the observed brightness the quantity of zodiacal light is the same for all areas.

It is, of course, desirable to use a formula which has a physical basis. We shall, therefore, try the function resulting from Seeliger's theory of the zodiacal glow,[1] which supposes the zodiacal light to be due to a reflection of sunlight by small particles of meteoric matter. The cloud of particles is symmetrical relatively to a plane which contains the axis of the zodiacal light and extends equally far in all directions in this plane. Seeliger found that on this hypothesis the intensity of zodiacal light depends in the following way on the density D of the meteoric matter and the law of phase $f(a)$:

$$\frac{H}{J}=\frac{C}{\sin\psi}\int_{\tau}^{\psi} D(x,y)\, f(a)da \qquad (1)$$

[1] "Ueber kosmische Staubmassen und das Zodiacallicht," *Sitzungsberichte der mathem. phys. Classe der kgl. bayer. Akademie der Wissenschaften*, **31**, Heft 3, 1901.

where

H = surface brightness of the zodiacal light

J = surface brightness of the sun

α = angle at the reflecting particle formed by the lines toward the sun and the earth. (See Fig. 1, where S represents the sun, O the observer, P a reflecting particle, and the plane SOP' the ecliptic. PP' is perpendicular to this plane)

ψ = Angle SOQ in Fig. 1 = supplement of the angle at the observer between the lines toward the sun and the particle.

τ = value of α at the exterior limit of the cloud

x and y = co-ordinates of the particle relative to the sun in a plane perpendicular to the ecliptic

$D(x, y)$ = density of the cloud as a function of the co-ordinates x and y

$f(\alpha)$ = function representing the dependence of the quantity of reflected light on the phase angle α

C = a constant

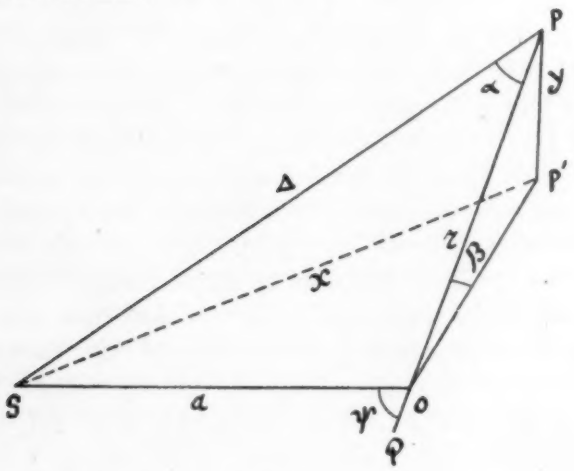

FIG. 1

If the function $f(\alpha)$ were known from terrestrial experiments, we might find the distribution of the density D by means of equations of the form (1), where the left-hand member is the observed quantity. This, however, is not the case; various laws have been

proposed,[1] but it is at present impossible to decide which of these is the right one.

Our observations, however, afford the material for a determination of the law of phases for small values of α, for in the neighborhood of the counterglow the variation of the zodiacal light with the position of the area depends almost exclusively on the law of phases, whereas the adopted density distribution is of little importance. This is clear a priori and has, moreover, been demonstrated by Searle.[2] The law of phases for small values of α can, therefore, be determined from the observed brightness of the zodiacal light near the counterglow. After some trials I found the fractions given in the third column of Table V. For large values of α the law of phases was made to agree approximately with the form of Lommel and Seeliger. Table V shows that my data yield a larger decrease of the brightness with the phase angle for small values of α than does Seeliger's formula. This is in agreement with the investigations made on the asteroids.[3] The adopted values of $f(\alpha)$ in the third column of Table V can be represented by the formula:

$$f(\alpha) = +0.655 - 0.711 \sin \alpha + 0.345 \cos \alpha + 0.406 \sin^2 \alpha \qquad (2)$$

TABLE V

THE LAW OF PHASES

α	Lommel and Seeliger	Adopted	α	Lommel and Seeliger	Adopted
0°	1.00	1.00	70°	0.54	0.46
10	.98	.88	90	.38	.35
20	.93	.78	110	.24	.23
30	.86	.70	130	.12	.13
50	0.70	0.57	140*	0.08	0.10

* Phase angles larger than 140° do not occur in the present investigation.

[1] Müller, *Die Photometrie der Gestirne*, pp. 58–62, 1897. See also Otto Frh. v.u.z. Aufsess, "Experimentelle Untersuchungen über den Einfluss der Phase und der Rotation auf die Helligkeit vor Kugeln und beliebig gestaltene Körpern," *Astronomische Abhandlungen als Ergänzungshefte zu den Astronomischen Nachrichten*, No. 17, 1910.

[2] *Harvard Annals*, **19**, 233, 1893.

[3] Müller, *op. cit.*, p. 378.

Various formulae have been tried for the density, which is to be determined from the condition that the zodiacal brightness, computed by means of (1), should agree with the observations. I first considered only the measures in the ecliptic for various longitudes, but later the higher latitudes were also taken into account. It was found that the formula

$$\text{Density} = D_0(1 - 0.176\, x^2 - 1.406\, y^2) \qquad (3)$$

satisfies the observations.

The brightness computed by means of formulae (1), (2), and (3) has been compared with the observed values in Table IV. The residuals O−C are all small. These considerations show that the present material at least does not contradict Seeliger's theory of the zodiacal light, although our lack of knowledge of the law of phases makes it impossible to put the theory to a severe test.

Formulae (1), (2), and (3), however, have been derived mainly for another purpose, viz., the computation of the zodiacal light at points of the sky for which no direct observations are available. The results of this computation have been given in Table VI, which represents the total amount of zodiacal light for different values of the latitude and longitude relative to the sun. It will be remembered that the values on which the table is based have been derived by increasing the excess of zodiacal light over a certain mean value by +0.069, and that this latter figure is somewhat uncertain on account of the hypothesis underlying its derivation. The same holds for the data of Table VI, the difference between any two values of the zodiacal light being more trustworthy than the values themselves. Fortunately it is the differences, in the main, which are needed for the reduction.

6. DETERMINATION OF THE STARLIGHT FOR GALACTIC LATITUDES LESS THAN 40° FROM THE OBSERVATIONS ALONG CIRCLES PARALLEL TO THE HORIZON

The method used to determine the starlight from series of measures parallel to the horizon is also illustrated by Table II. The first ten columns have already been explained. In the twelfth

column is given a correction for the excess of zodiacal light over a certain mean value. This has been derived by subtracting 0.069 from the figures of Table VI and reversing the sign of the difference

TABLE VI

COMPUTED VALUES OF THE ZODIACAL LIGHT

(Unit=a star of mag. 1.00, Harvard Visual Scale)

Long. from Sun	Latitude (β)									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
0°	.....	.....	.....	.....	0.112	0.077	0.060	0.050	0.046	0.045
10	.....	.....	.....	.....	.110	.076	.060	.050	.046	.045
20	.....	.....	.....	.....	.103	.076	.060	.050	.046	.045
30	.....	.....	.....	.....	.097	.073	.059	.049	.046	.045
40	0.325	0.258	0.191	0.132	.090	.072	.059	.049	.046	.045
50	.247	.206	.161	.113	.086	.071	.058	.049	.046	.045
60	.184	.168	.138	.105	.084	.069	.057	.049	.046	.045
70	.149	.139	.119	.097	.079	.066	.055	.049	.046	.045
80	.127	.119	.103	.087	.073	.063	.053	.048	.046	.045
90	.109	.102	.090	.079	.068	.061	.053	.047	.046	.045
100	.098	.090	.081	.075	.065	.060	.052	.047	.046	.045
110	.090	.084	.076	.071	.064	.059	.052	.047	.046	.045
120	.086	.080	.073	.068	.061	.057	.052	.047	.046	.045
130	.082	.077	.070	.066	.060	.057	.052	.047	.046	.045
140	.081	.078	.070	.066	.060	.056	.051	.047	.046	.045
150	.081	.078	.071	.066	.060	.056	.051	.046	.045	.045
160	.082	.079	.072	.066	.060	.056	.051	.046	.045	.045
170	.085	.082	.074	.067	.060	.056	.050	.046	.045	.045
180	0.097	0.090	0.075	0.067	0.060	0.055	0.050	0.046	0.045	0.045

thus found. These corrections α have been applied to the figures of the tenth and eleventh columns, thus giving the results in the thirteenth and fourteenth columns, respectively. The areas in small and large galactic latitudes have been treated separately, because for the former the amount of the starlight is unknown.

The thirteenth column thus contains the observed brightness corrected for direct starlight (s), scattered starlight (s'), and excess of zodiacal light (z), whereas the figures in the fourteenth column have been corrected for the last two quantities only. The values of $B^z_{s',s}$ and $B^z_{s'}$ include an amount of zodiacal light which is the same for all areas. The quantities $B^z_{s',s}$ appear to be independent of the azimuth. We therefore form their mean, for which the corresponding galactic latitude exceeds 40°, and subtract the result

from each value of $B^z_{s'}$ in the fourteenth column. The differences inserted in the fifteenth column represent the starlight per square degree, reduced for atmospheric absorption to the zenith.

Most of the series were treated in this way. In only a few cases was the number of observations in galactic latitudes greater than 40° too small to apply this method. In such cases I computed the mean value of $B^z_{s',s}$ from the measures whose galactic latitudes exceed 20°. The starlight between 20° and 40° required for the reduction of these cases was obtained from all other series parallel to the horizon which include a sufficiently large number of observations in the higher galactic latitudes. The results are averaged in Table VII, the probable errors being derived from the internal agreement of the observations.

TABLE VII

TOTAL LIGHT PER SQUARE DEGREE OF ALL STARS FAINTER THAN MAG. 5.5 DERIVED FROM SERIES PARALLEL TO THE HORIZON

(Unit = a star of mag. 1.00, Harvard Visual Scale)

GAL. LAT.		STARLIGHT	PROB. ERROR
Limits	Mean		
0° to 9°	4°.2	0.068	±0.003
10 to 19	14.1	.042	.002
20 to 29	25.8	.032	.002
30 to 39	35.5	0.013	±0.002

7. DEPENDENCE OF THE EARTHLIGHT, CORRECTED FOR ZODIACAL GLOW, ON THE ZENITH DISTANCE. PROBABILITY OF A PERPETUAL AURORA

We pass to the consideration of the series of observations perpendicular to the horizon. The variation of the earthlight with the zenith distance will be investigated first. This can be done only for areas whose galactic latitudes exceed 40°, where the starlight is supposed to be known. An example of the method employed is given in Table VIII.

The first nine columns of Table VIII contain the same quantities as the corresponding columns in Table II. The tenth column

TABLE VIII

Reduction of a Series of Observations Perpendicular to the Horizon

No.	b	β	λ	z	A	B	$B_{s'}$	$B_{s',s}$ $b>40°$	Corr.* Zod. L.	$B^z_{s',s}$ $b>40°$	$B^z_{s'}$ $b<40°$	Corr. Sc.E.L.	$B^z_{s',e}$ $b>40°$	$B^z_{s',e}$ $b<40°$	Starlight	
															Uncorr.	Corr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
194.....	+72°	+41°	+ 30°	73°	132°	0.158	0.147	0.141	−0.018	0.123	.....	−0.041	0.082	.....	.....	.....
195.....	+66	+46	+ 24	71	138	.139	.129	.122	− .012	.110	.....	− .039	.071	.....	.....	.....
196.....	+62	+56	+ 39	59	136	.133	.125	.117	+ .004	.121	.....	− .028	.093	.....	.....	.....
197.....	+55	+63	+ 49	51	137	.115	.108	.099	+ .013	.112	.....	− .024	.088	.....	.....	.....
198.....	+45	+70	+ 76	39	135	.115	.109	0.098	+ .020	0.118	.....	− .020	0.098	.....	.....	.....
199.....	+36	+71	+108	29	134	.115	.109	.....	+ .022	.....	0.131	− .018	.....	0.113	0.020	0.020
200.....	+25	+70	+141	19	138	.124	.119	.....	+ .022	.....	.141	− .016	.....	.125	.032	.032
201.....	+18	+68	+165	16	155	0.138	0.133	.....	+0.022	.....	0.155	−0.016	.....	0.139	0.046	0.046

* This correction has been reduced to the zenith distance z for atmospheric absorption.

gives the correction for the excess of zodiacal light derived in the same way as in section 6. The corrected values in the eleventh column thus contain a constant quantity of zodiacal light, the scattered earthlight, and, in addition, any light of unknown origin. In the thirteenth column is given a correction for scattered earthlight, derived by means of Abbot's observations already referred to and the total amount of earthlight known from the present investigation. The fourteenth column, in which the scattered earthlight is subtracted from the data in the eleventh column, thus contains a quantity of zodiacal light constant over the whole sky and perhaps some light produced by unknown sources of illumination. In order to find whether there are such sources, I have investigated the variation of the values of $B^{z;e}_{s',s}$ with the zenith distance. The data are collected in Table IX; all measures similar to those of Table VIII, for which the galactic latitude exceeds 40°, have been used.

TABLE IX

MEAN BRIGHTNESS CORRECTED FOR STARLIGHT, EXCESS OF ZODIACAL LIGHT, AND SCATTERED EARTHLIGHT $\left(B^{z;e}_{s',s}\right)$ AS A FUNCTION OF THE ZENITH DISTANCE

Limits z	Mean z	$B^{z;e}_{s',s}$	Prob. Error	No.	Weight	$B^{z;e}_{s',s}$ in Zenith	Comp.
10° to 29°	24°	0.085	±0.004	8	1.6	0.087	0.087
30 to 39	36	.084	.003	11	2.2	.087	.089
40 to 49	45	.090	.003	14	1.4	.095	.092
50 to 59	54	.086	.003	20	2.0	.096	.096
60 to 69	64	.086	.003	21	2.1	.104	.103
70 to 77	73	0.077	±0.005	13	0.6	0.111	0.116

The seventh column of Table IX gives the values of $B^{z;e}_{s',s}$ corrected for atmospheric absorption. If these values included nothing but a constant quantity of zodiacal light for the whole sky, they would be independent of the zenith distance. Since they increase with increasing zenith distance, there must exist some other source of illumination whose intensity is greatest near the horizon. This conclusion is strengthened by the fact that Slipher has photographed the green auroral line in the spectrum of the background of the sky.[1] A study by Störmer of the auroras

[1] *Popular Astronomy*, **25**, 274, 1916.

occurring in 1913 showed that they originated in a comparatively shallow layer at a great height above the earth,[1] whence we conclude that the sky light due to this source should vary directly as the thickness of the auroral layer in the direction of the line of sight, i.e., as the secant of the zenith distance. This holds only for zenith distances smaller than say 70°.

We may therefore write

$$B^{z;e}_{s';s} = a + p \sec z,$$

where a represents the constant quantity of zodiacal light and $p \sec z$ the auroral brightness. A least-squares solution of the data of Table IX gives

$$a = +0.076 \qquad p = +0.0113.$$

The values computed with these constants are given in the last column of Table IX.

Other solutions were also made, which allow for the deviation of the thickness of the auroral layer from the secant law. Taking these into consideration the final value is $a = +0.072$. This differs little from the value $+0.069$, adopted in section 5 in accordance with a provisional solution. It is to be noted that the constant quantity of zodiacal light thus determined, which is the remainder after applying the correction indicated above by a, has been computed on the hypothesis that the only source of light, except the zodiacal glow, is the aurora borealis, and that the illumination contributed by the aurora varies as the secant of the zenith distance.

8. DETERMINATION OF THE STARLIGHT FOR GALACTIC LATITUDES LESS THAN 40° FROM OBSERVATIONS ALONG CIRCLES PERPENDICULAR TO THE HORIZON

From the third column of Table IX it appears that the observed brightness corrected for starlight, excess of zodiacal light, and scattered earthlight is independent of the zenith distance down to $z = 70°$. This fact has been used as follows to determine the starlight near the galaxy.

[1] *Astrophysical Journal*, **43**, 243, 1916.

The fifteenth column of Table VIII gives the observed brightness at low galactic latitudes, corrected for scattered starlight, excess of zodiacal light, and scattered earthlight,[1] and thus includes exactly the same sources of illumination as the values in the fourteenth column, plus the direct starlight. It has been stated already that the quantities in this column are independent of the zenith distance down to $z=70°$. In order to find the starlight at lower galactic latitudes, we therefore need only diminish the figures in the fifteenth column by the average value of the fourteenth column for $z<70°$. This has been done in the case of Table VIII. The values of the starlight inserted in the sixteenth column have been corrected for atmospheric absorption and appear in the seventeenth column.

All series of measures perpendicular to the horizon have been treated in this way, except that in a certain number of cases the mean value of $B^{z}_{s}, ^{e}_{s}$ has been computed from all areas whose galactic latitudes exceed 30°, or even 15°, because the series in question do not include a sufficient number of observations in galactic latitudes greater than 40° to afford a reliable result. The starlight for galactic latitudes 15° to 40° thus required for the derivation of the values of $B^{z}_{s}, ^{e}_{s}$ was found from the series perpendicular to the horizon which do contain a sufficient number of observations between 15° and 40° as well as above 40°. The results are given in Table X.

TABLE X

TOTAL LIGHT PER SQUARE DEGREE OF ALL STARS FAINTER THAN MAG. 5.5 DERIVED FROM THE SERIES OF OBSERVATIONS PERPENDICULAR TO THE HORIZON

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Limits b	Mean b	Starlight	Prob. Error
0° to 9°	4°.5	0.081	±0.003
10 to 19	14	.056	.002
20 to 29	24	.030	.001
30 to 39	35	0.016	±0.001

[1] The seventh column gives the observed brightness; in the eighth column this brightness has been corrected for scattered starlight; in the twelfth column the excess of zodiacal light has been subtracted and in the fifteenth a correction for scattered earthlight has been applied.

9. FINAL RESULT FOR THE AMOUNT OF STARLIGHT AS A FUNCTION OF THE GALACTIC LATITUDE. COMPARISON WITH OTHER AUTHORITIES. TOTAL AMOUNT OF STARLIGHT OVER THE WHOLE SKY

Comparing Tables VII and X we see that the values of the starlight derived from the series parallel and perpendicular to the horizon agree pretty well. The difference between the two series of results for the region of the Milky Way is at least partially real, for Easton's observations[1] give even larger values for the starlight of those galactic areas occurring in the series perpendicular to the horizon than do my own measures.

The results of Tables VII and X have been combined with equal weight in Table XI. The light of the stars brighter than magnitude 5.5, which is not included in my measures, has also been

TABLE XI

TOTAL AMOUNT OF STARLIGHT PER SQUARE DEGREE

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Gal. Lat.	This Paper	Yntema	*Gron.*, 27	*Gron.*, 18
0°	0.080	0.088	0.055	0.186
10	.065	.055	.043	.118
20	.044	.033	.027	.055
30	.026	.024	.020	.032
40	.015	.020	.015	.022
50	.014	.018	.014	.018
60	.012	.015	.012	.015
70	.011	.013	.011	.013
80	.010	.013	.010	.012
90	0.010	0.012	0.010	0.012

added. In addition Yntema's values[2] and those computed by means of the number of stars in *Groningen Publications*, No. 27, Table V, and *Groningen Publications*, No. 18, are also given. The agreement with Yntema's values is good; but the enormous difference between Yntema and myself, on the one hand, and *Groningen Publications*, No. 18, on the other, shows that the magnitude scale

[1] *Verhandelingen der Koninklyke Akademie van Wetenschappen te Amsterdam*, Eerste sectie, deel VIII, No. 3, 1903.

[2] *Groningen Publications*, No. 22, p. 31, 1909.

used for the latter is probably in error. The same conclusion was arrived at from other evidence in *Groningen Publications*, No. 27.

The total amount of light received from all the stars in both hemispheres as computed from the present material is equal to 1440 stars of magnitude 1.00, Harvard Visual Scale.

10. ACCIDENTAL AND SYSTEMATIC ERRORS

The mean of two measures of the brightness of the sky, made by moving the lamp in opposite directions, may be considered as a single observation. Its probable error, computed from the differences between the values found for the same area, is found to be 0.7 per cent of the amount. This does not contain the uncertainty due to errors in the magnitude of the artificial star. Including these, the probable error of a complete measure is about 2 per cent of the amount.

It is practically certain that the relative brightness of different areas, as measured in the present paper, is free from systematic error. In addition I have taken all possible precautions to obtain the correct value for the absolute amount of light received from one square degree at the North Pole. Nevertheless my value differs considerably from that found by Newcomb and Burns, which is as follows:

TABLE XII

VALUES OF THE EXTRA GALACTIC SKY BRIGHTNESS FOUND BY DIFFERENT OBSERVERS

Authority	Brightness per Square Degree
Newcomb, *Astrophysical Journal*, **14**, 297	0.029
Burns, *ibid.*, **16**, 166	0.050
Abbot, *Astronomical Journal*, **27**, 20	0.075
Yntema, *Groningen Publications*, No. 22	0.140
van Rhijn, present paper	0.130

Burns does not give all the details necessary to check his method of measurement and reduction. I do not feel sure that his results are free from objection. He compares the brightness of the sky with the extra-focal image of a star of known magnitude, the sky being observed with one eye and the extra-focal image, as seen through the telescope, with the other. I do not think that accurate

results can be expected if, as in this case, the sources of brightness compared are not in contact. The internal agreement of Burns's observations is, moreover, rather poor.[1] This method seems, therefore, inferior to that used in the present paper.

Abbot and Yntema used the same instrument and method as I myself have used. In view of the difficulties affecting all kinds of photometric measurements, further determinations of the absolute amount of the sky-brightness are very desirable. I have thought of various methods which may be used, and hope in the near future to give some of them a trial.

GRONINGEN ASTRONOMICAL LABORATORY
September 1919

[1] *Op. cit.*

PHOTOGRAPHIC EFFECTIVE WAVE-LENGTHS OF NEBULAE AND CLUSTERS

SECOND PAPER

By KNUT LUNDMARK and BERTIL LINDBLAD

Since our first results concerning the photographic effective wave-lengths of some bright spiral nebulae and globular clusters were published,[1] exposures of nebulae with the twin Zeiss-Heyde astrograph ($a = 15$ cm) of the Observatory of Upsala have been continued during the years 1917 and 1918 in order to derive color-equivalents for a greater number of objects. The general principles of measurement and reduction are given in our first paper mentioned. The wire grating in front of the objective I is the same in the new investigation, but here we have also used objective II, supplied with a grating of somewhat larger constant. Though for fixed stars the images given by objective II are inferior in quality to those given by objective I, the former objective has yielded useful results here, especially for some spiral nebulae. The great advantage of getting two images in the same exposure need not be pointed out, when the exposures in question are extended over several hours; in some cases, for instance for the spiral nebula M 51 and large globular clusters, it is very valuable to be able to put the grating spectra of the two objectives into different angles of position in order to discriminate more easily between the spectra and details of the object situated far from the center. Generally the two gratings are placed with the threads in right angles to one another, in the directions of right ascension and declination respectively. A rather large number of those different kinds which are included in the common name of nebulae has been photographed, but for a part of these objects we have not been able to obtain measurable images. In the estimation of the probability of securing a certain object the magnitudes of Holetschek[2] have been very

[1] *Astrophysical Journal*, **46**, 206, 1917; *Astronomische Nachrichten*, **205**, 161, 1917.

[2] *Annalen der k.k. Universitäts-Sternwarte in Wien*, Band **20**, 1907.

useful, but the chief condition for a successful result is that the increase of brightness toward the center is rather rapid, except for those objects the surface brightness of which is very great. This is the case especially for some planetary nebulae, which give very definite results. Their effective wave-lengths are, moreover, very characteristic. Generally, however, an object with a well-defined nucleus, i.e., most nearly resembling a fixed star, gives the best results. In this way the nebula N.G.C. 5473, of magnitude $10^{M}.7$ in Holetschek's table, is well measurable, while many objects of magnitude $<9^{M}$ are impossible with the same time of exposure. Further, the atmospheric conditions sometimes have not permitted us to expose an object for all the time which was thought necessary in our estimations; sometimes again the images have been perceptibly deformed, especially near the margins of the plate.

It is a great pleasure to us to express here our gratitude to Mr. Sten Asklöf and Mr. Axel Corlin for the great interest they have had in our work, especially by taking a very considerable part in the long and tiresome exposures.

Table I contains a list of the plates taken for our purpose. The first and second columns contain the date of the plate and the sidereal time of beginning and ending the exposure, the third contains the times of exposure, the fourth the observers, the fifth the objects photographed.

The constant of the grating in front of objective II was measured by Lundmark. The positions of ninety-seven consecutive threads were measured on the millimeter scale of the Hartmann microphotometer, in which the grating was put into the frame of plates. Groups of five threads were also measured in the Repsold measuring machine. The result was: $c=1.5258$ mm. The focal length of the same objective was found by Lundmark to be 1478.7 mm. Thus the effective wave-length is given by the formula

$$\lambda_{eff}=515.93 \cdot s\,\mu\mu.$$

where s is the distance, expressed in mm, between the centers of density of the spectra. For the grating of objective I, Lundmark got the result $c_{I}=1.3413$ mm in good agreement with that previously derived, $c_{II}=1.3422$. Because the second value has been

TABLE I

Date	Sidereal Time	Exposure	Observers	Object	Remarks
1917 Aug. 27..	21^h20^m9— 0^h44^m9	3^h24^m	Lk., Ld.	N.G.C. 598	Cloudy
Sept. 11..	21 50.1— 0 50.1	3 0	Lk., Ld.	598	Hazy
18..		5 0		6946	Sky good
19..	21 26.0— 1 29.5	15 45 2 20	Lk.	205, 221, 224	Sky good, strong wind
24..	19 56.5—20 12.5	5	Lk., Ld.	7662	
24..	22 57.0—23 46.5	49.3	Lk., Ld.	7027	
Oct. 2..	21 20.5—22 13.5	10 16 24	Lk.	6826	Sky good
5..	20 39.0—21 48.0	5 15 45	Lk., Cn.	7009	Unsteady in the air
6..	23 40.7— 2 5.7	5 19 2 0	Lk., Cn.	6503, 6543	
8..	0 27.7— 1 13.2	45.5	Lk., Ld., Cn.	650–651	
11..	21 3.1—23 47.6	2 44.5	Lk., Ld., Cn.	6229	Hazy at the end of the exposure
15..	21 3.1— 0 15.6	3 10	Lk., Ld.	6720	
28..	1 25.5— 2 38.0	3 5 15 45	Lk., Cn.	1514	The air unsteady at the end of the exposure
Nov. 3..	21 34.8— 1 29.8	2 15	Lk., Cn.	7023	
11..	1 48.0— 2 22.0	34	Lk., Ld., Af.	205, 221, 224	Clouds interrupted the exposure
15..	21 27.1— 5 27.1	8 0	Lk., Ld., Af., Cn.	598	
18..	21 43.2— 1 13.2	3 30	Lk., Cn.	7814	Air unsteady
	2 6.2— 5 6.2	3 0	Lk., Cn.	1068	Air unsteady
20..	23 19.3— 1 5.7	1 36.4	Lk., Ld., Af., Cn.	205, 221, 224	
Dec. 4..	22 32.9— 2 35.9	4 3	Lk., Ld., Af.	650–651	Sky good
14..	23 45.2— 0 30.2	45	Lk.	650–651	Sky very fine

1918	Jan.	6..	0 56.0— 3 57.0	3 1	Lk.	N.G.C. 628	Temperature −21° Celsius
		13..	2 48.4— 6 3.4	3 15	Lk., Af.	2403	
	Feb.	1..	3 9.8— 9 32.8	3 0 3 0	Lk., Af.	2245, 2261	The plates were taken for stellar-statistical purposes
		15..	4 43.1— 9 15.1	4 32	Lk., Ld., Cn.	3031, 3034, 3077	
		17..	6 24.4— 9 46.4	3 22	Lk., Ld.	2415	The plate taken for obtaining λ_{eff} of Wolf's object
		28..	7 4.4—10 4.4	3 0	Lk., Ld., Af., Cn.	2841	Sky very good
	Mar.	2..	6 43.8—13 43.8	7 0	Lk., Af.	3377, 3379, 3384, 3412	
		3..	6 30.8—10 0.8	3 30	Lk., Af., Cn.	2903, 5	Sky very fine
		5..	10 16.8—12 16.8	2 0	Lk., Ld., Af.	4736	
		6..	8 25.1—15 48.8	6 3	Lk., Ld., Af.	$\alpha = 12^h25^m$; $\delta = +13°$	
		7..	7 36.4—11 5.8	3 30	Lk., Ld., Af.	3623, 3627	
		9..	7 6.8—15 38.9	3 30		3587	
		10..	7 39.0—13 9.8	5 30	Lk., Af.	$\alpha = 12^h30^m$; $\delta = +12°$	
		17..	8 34.2—15 49.9	7 0	Lk.	3587	Sky very good
		23..	10 46.3—11 48.3	30 30	Lk., Ld.	4736	
	April	4..	10 57.7—11 29.2	31.5	Ld.	$\alpha = 12^h28^m$; $\delta = +27°.5$	
		11..	11 21.2—15 52.7	5 31	Lk., Bd., Cn.	5422, 5457–8, 5473, 5485, etc.	
		16..	11 4.0—16 35.0	4 31	Lk., Ld., Af., Cn.	$\alpha = 12^h28^m$; $\delta = +27°.5$	Hazy at the end of the exposure
		25..	13 17.4—14 17.7	36 15 5	Ld., Cn.	6205	
		26..	12 7.8—14 7.8	2 0	Lk., Ld.	6205	
	May	2..	12 59.0—16 19.0	3 20	Lk., Ld., Cn.	5194, 5195	
		3..	14 13.0—15 13.0	1 0	Lk., Cn.	6341	
			15 37.7—16 37.7	1 0		6218, 6254	

* Lk. = Lundmark; Ld. = Lindblad; Af. = Asklöf; Cn. = Corlin; Bd. = Bergstrand.

adopted for some previous work it has also been used here. Thus we get the wave-lengths for objective I, the focal length being 1485.5 mm, by the formula

$$\lambda_{eff}=451.74\cdot s\,\mu\mu.$$

The values of λ_{eff} derived in this way need some corrections. At first there is a systematic difference between the results from objective I and objective II. We have found on an average from our measures on the nebulae,

$$\lambda_{II}-\lambda_{I}=7\,\mu\mu.$$

The values for objective II are reduced to the system of objective I by applying the correction $-7\,\mu\mu$. This value of the correction is confirmed by measures on spectra of twenty fixed stars effected by Mr. Ossian Vallin. By reason of the difficulties of international communications during the war we have not been able to use here the sensitive Imperial plates employed for our first paper. Instead we used at first Hauff, Ultrarapid, size 9×12 cm; then we found Wellington Press plate better for our purpose on account of a somewhat greater sensitiveness. For most objects in this investigation we have therefore used this sort of plate, of the size 9×12 cm. For stars of late type the correction from Wellington to Imperial has been found to be about $-3\,\mu\mu$. The correction is smaller for early types; further, this value of the correction is valid only for a certain rather strong intensity of stellar images. By comparing the results below for the effective wave-lengths of faint images of Saturn, derived from a Wellington plate with the corresponding values for an Imperial plate in our previous paper, we find the average correction $-1.3\,\mu\mu$ for intensities of images nearly corresponding to those of the nebulae. The difference is larger for the stronger images. We have applied the correction $-1.3\mu\mu$ to wave-lengths measured on Wellington plates and exceeding 420 $\mu\mu$. Other values of the effective wave-lengths we have left unchanged. For Hauff plates the correction to the Imperial system thus is assumed to be negligible. A possible error introduced by this assumption cannot be of any serious importance.

A matter of importance in this connection is the variation of the effective wave-length with the intensity of image. We have

tried to re-examine this effect by taking several exposures of Mars and Saturn on the same plate; the results are shown in Table II. The faintest images of Mars are somewhat stronger than those of exposure 3 sec. of Saturn. The shortest exposures of Saturn correspond to very faint images, even fainter than most of the nebular images measured. The change of λ_{eff} with the intensity seems to be in agreement with the results derived by Lindblad[1]

TABLE II

PLATE TAKEN MARCH 23, 1917

SIDEREAL TIME 11^h57^m—12^h26^m

SATURN					MARS				
Exposure	Number of Images	λ_c			Exposure	Number of Images	λ_c		
		Lundmark	Lindblad	Mean			Lundmark	Lindblad	Mean
s		μμ	μμ	μμ	s		μμ	μμ	μμ
⅓	2	433.8	432.3	433.0					
1	3	34.5	32.9	33.7					
3	2	34.7	36.1	35.4	⅓	3	434.2	435.6	434.9
10	2	33.4	35.0	34.2	1	3	34.7	32.8	33.8
25	1	434.3	434.9	434.6	3	2	33.5	32.5	33.0
					10	2	32.9	31.2	32.1
					30	1	35.5	35.1	35.3
					60	1	438.1	444.9	441.5

for stars of late type. A minimum of the effective wave-length for a certain intensity is rather conspicuous for the two planets; on the whole the change of λ_{eff} is very small. For the early-type stars, however, with small effective wave-lengths, the effect has been found stronger. We have no sure direct experience of how the effect may appear for a nebula of low λ_{eff}; for the planetary nebulae in Table III, however, the effect does not seem to be very serious. For spiral nebulae and clusters the effective wave-lengths are on an average rather large; the intensities of image may always be assumed to lie between those corresponding to the exposures, ⅓ sec. and 10 sec. of Saturn. Thus the effect may be neglected for those objects, and consequently we may without great error apply the relations found for faint stellar images between the effective

[1] *Arkiv för Matematik, Astronomi och Fysik*, **13**, No. 26, Stockholm, 1918.

wave-lengths on Imperial plates and the spectral type, provided only that the relation between the appearance of characteristic spectral lines and the distribution of intensity in the continuous spectrum is really the same for the nebulae as for the fixed stars. That such an assumption is legitimate, at least for the photographic part of the spectrum, is shown by the coincidence of our computed spectral types with those determined from the spectral lines by Fath and by Wolf. The relation between the effective wave-lengths for faint stellar images and the spectral types of the usual Harvard sequence are given below according to the results of Lindblad.[1]

Spectral type:	B	A	F	G	K	M
λ_{eff} :	408	413	420	428	433	444 $\mu\mu$

For the three last types the values of λ_e are valid for the "giant" series. For "dwarfs" the values come out some units smaller. The discontinuous spectra of some objects do not now permit us to apply the usual sequence of spectral types, thus to assign, for instance, the spectral type B or A to a planetary nebula. Such objects we refer to a common class of gaseous nebulae, type P.[2] The planetary nebulae we know to be intimately connected with the Wolf-Rayet stars, type O, which have nearly the same color as the helium stars, type B; it is rather interesting that the distribution of intensity in the discontinuous spectra of the planetaries really agrees with the color of these two spectral types. Another interesting fact is that the novae seem to be attached to this group of objects. The effective wave-length of Nova Aquilae 3 was found by Lundmark[3] to be very nearly that of the planetaries during the time when the continuous spectrum weakened and was successively overwhelmed by the bright lines.

Objects with effective wave-lengths below that of type Ao we may probably always refer to the nebular class P; for somewhat greater wave-lengths there may be confusion between planetary nebulae on one hand and clusters or spiral nebulae of early types

[1] *Loc. cit.*

[2] *Harvard Annals*, **76**, 20, 1916.

[3] *Astronomische Nachrichten*, **205**, 73, 1918.

on the other. Here we may, strictly speaking, only assign correct spectral notations if we know in advance the character of the object. Spiral nebulae of early type, however, are certainly very exceptional, the clusters probably not departing widely from type F; on the other hand the greatest effective wave-length for a planetary nebula measured is 416 $\mu\mu$. The annular nebula in Lyra, another type of gaseous nebulae, has an exceptionally small effective wave-length. The separation of different nebular types by the effective wave-length thus is rather good, and it seems as if we may with a fair precision predict the character of an object by means of its effective wave-length.

The results of the measurements are shown in Table III. The first three columns contain the name of the object, its right ascension and declination, the fourth the magnitude according to Holetschek, the next three columns the time of exposure, the date of the plate, and the sort of plates used (I = Imperial, H = Hauff, W = Wellington). The next following columns contain the corrected effective wave-length λ_c, derived from cameras I and II respectively. Here we have also taken up the values of λ_c in our first paper. Generally the nebulae are measured by both of us, and the differences between our values, Lk. − Ld., are tabulated in the columns headed $\Delta\lambda$. The next column contains the adopted value of λ_c, derived as a weighted mean of the values in the preceding columns, where the weights are given according to various circumstances, the intensity and symmetry of the images, and the magnitude of the differences $\Delta\lambda$. Then we have tabulated the relative weights assigned to the final values; they are estimated by considering the number and weights of the separate images used. The next column contains the color-indices in the system of *Harvard Annals*, **80,** where the color-indices for types A and K are put equal to $0^{M}.00$ and $1^{M}.00$, respectively. The color-indices were computed with the spectral type in the sequence B–M as argument, derived from the effective wave-lengths as if the objects were all fixed stars. This color-index is thus a direct translation of λ_c into another color-system, but it does not mean that the color-indices for the objects would necessarily be found to be those in the table, if determined in the usual way. This conclusion is obvious for the

TABLE III

Object N.G.C.	α_{1900}	δ_{1900}	H	Exposure	Date	Sort of Plates†	λ_c I	λ_c $\Delta\lambda$ (Lk.−Ld.)	λ_c II	λ_c $\Delta\lambda$ (Lk.−Ld.)	λ_c Adopted	λ_c Weight	Color-Index	Spectrum Inferred	Spectrum Observed	Class of Objects
221*.....	$0^h37^m.3$	+40° 19′	$8^M.7$	140m	da. mo. yr.		μμ	μμ	μμ	μμ	μμ					
				45	19. 9. 17	I	432.4	+0.1	.....	.....	.....	.....	.....	.....	.....	
				15	19. 9. 17	I	32.4	−0.5	.....	.....	.....	.....	.....	.....	.....	
224*.....	0 37.3	+40 44	5.0	96.4	19. 9. 17	I	31.5	+2.9	.....	.....	.....	.....	.....	.....	.....	Spiral
				140	20. 11. 17	H	30.6	+0.8	.....	.....	432	5	+0M91	G8		
				45	19. 9. 17	I	30.4	+5.5	.....	.....						
1068*.....	2 37.6	− 0 26	8.7	96.4	19. 9. 17	I	31.0		.....	.....						
1514*.....	4 3.0	+30 31	.....	180	20. 11. 17	H	28.2	+6.3	.....	.....	430	2	+0.74	G4	G–K	Spiral
				45	18. 11. 17	H	20.2	.....	415.3	+1.0	420	1	+0.28	Fo	G	
2245*.....	6 27.2	+10 14	9.5	15	28. 10. 17	H	11.8	−2.7	.....	.....						
2261*.....	6 33.7	+ 8 49	9.7	180	28. 10. 17	H	7.7	.....	.....	.....	410	2	−0.14	P		
2841*.....	9 15.1	+51 24	9.1	180	1. 2. 18	H	0.5	.....	.....	.....	422	1	+0.34	F2	G?	Cometic
2903*.....	26.5	+21 56	9.0	180	1. 2. 18	H	21.6	.....	.....	.....	422	1	+0.34	F2	G?	Cometic
3031.....	47.3	+69 32	8.0	210	28. 2. 18	W	33.4	+1.4	.....	.....	433	3	+1.00	Ko	.....	Spiral
				129	3. 3. 18	W	23.6	−0.4	.....	.....	424	3	+0.42	F5	.....	Spiral
				274	16. 2. 17	I	35.8	+0.9	.....	.....						Spiral
3034*.....	47.6	+70 10	8.8	272	22. 2. 17	I	35.7	+0.1	.....	.....	.....	.....	.....	.....	.....	
				274	15. 2. 18	W	.....	.....	427.3	+3.4	433	3	+1.00	Ko	K	
3077*.....	9 55.3	+69 13	10.2	272	22. 2. 17	I	25.0	−2.0	.....	.....	.....	.....	.....	.....	.....	
				274	15. 2. 18	W	28.6	+2.3	18.6	+2.6	422	2	+0.34	F2	.....	Spiral?
3377*.....	10 42.4	+14 31	9.8	272	22. 2. 17	I	.....	.....	15.6	.....	424	1	+0.42	F5		
3379*.....	42.6	+13 6	9.1	420	15. 2. 18	W	32.8	−1.4	.....	.....	433	3	+1.00	Ko		
3384*.....	43.0	+13 9	9.5	420	2. 3. 18	W	32.1	+1.3	.....	.....	432	3	+0.91	G8		
3412*.....	10 45.6	+13 56	9.5	420	2. 3. 18	W	28.5	+3.3	.....	.....	428	3	+0.56	Go		
3587*.....	11 9.0	+55 34	9.1	420	2. 3. 18	W	26.8	−0.1	.....	.....	427	3	+0.53	F9		
				210	17. 3. 18	W	.....	.....	09.3	−2.8		2				
Anonyma*....	9.2	+55 31	13	420	9. 3. 18	W	10.1	+1.9	.....	.....	410	2	−0.14	O?	.....	Planetary
					17. 3. 18	W	14.3	+3.3	12.9	.....	414	2	+0.05	O?	.....	Star with nebulous rays
3613.....	12.8	+58 33	10.5	420	17. 3. 18	W	.....	.....	39.2	−3.6	439	2	+1.20	K6		
3619.....	13.6	+58 18	11.0	420	17. 3. 18	W	.....	.....	39.1	−4.9	439	1	+1.20	K6		
3623*.....	13.7	+13 38	8.9	210	7. 3. 18	W	34.0	−2.2	33.7	−0.5	434	4	+1.03	K1		
3627*.....	11 15.0	+13 32	8.6	210	7. 3. 18	W	20.7	−0.1	20.8	+1.6	421	4	+0.31	F1	.....	Spiral
4251.....	12 13.1	+28 44	9.5	271	11. 4. 18	W	29.4	.....	29.9	0.0	430	1	+0.74	G4	.....	Spiral
4278.....	15.1	+29 50	9.6	271	11. 4. 18	W	25.0	−0.6	21.1	+0.5	424	2	+0.42	F5		
4494.....	26.4	+26 20	9.2	271	11. 4. 18	W	28.3	−1.5	428.2	−3.4	428	2	+0.56	Go		
4725.....	45.5	+26 3	8.7	100	19. 4. 17	I	30.4	+3.3	.....	.....	430	1	+0.74	G4	G	Spiral
				100												

4736*.....	12 46.2	+41 40	7.7	105	15. 3. 17	I	26.7	+1.2	.....	.....	427	1	+0M53	F0	.....	Spiral
				105	15. 3. 17	I	32.9	+0.2	.....	.....	433	1	+1.00	K0		
				120	5. 3. 18	W	31.5	+3.6								
				30	23. 3. 18	I	33.0									
				30	23. 3. 18	I	431.3	.....	.....	.....	432	4	+0.91	G8	G	
4826.....	12 51.8	+22 13	8.6	100	19. 4. 19	I	433.8	−0.1	.....	.....	434	1	+1.03	K1	G	Spiral
				100												
5194.....	13 25.7	+47 43	8.4	180	27. 3. 19	I	30.7	+2.0								
						W	35.2	−3.6	440.9	−4.6	436	2	+1.11	K3	K	
5195.....	25.8	+47.47	8.6	180	27. 3. 19	I	26.3	0.0	.....	.....	.....	.....	.....	.....	.....	Spiral
						W	32.5	−3.2	35.0	−1.8	431	3	+0.82	G6	.....	
5272.....	37.6	+28 53	.....	.....	.....	I	25.1	+0.3	.....	.....	425	1	+0.45	F6	A–G	Globular cluster
5423.....	57.3	+55 39	10.2	300	11. 4. 18	I	.....	.....	26.7	−3.6	427	1	+0.53	F9		
5457.....	13 59.7	+54 50	9.2	300	11. 4. 18	I	30.9	+2.3	32.0	.....	431	2	+0.82	G6	.....	Spiral
5471.....	14 0.9	+54 53	.....	300	11. 4. 18	I	30.5	−2.9	.....	.....	431	1	+0.82	G6		
5473.....	14 1.2	+55 23	10.7	300	11. 4. 18	I	28.4	−2.2	429.2	−4.6	429	3	+0.64	G2		
5485.....	14 16.6	+57 11	11.0	300	11. 4. 18	I	30.3	−3.7	.....	.....	430	2	+0.74	G4		
5904.....	15 13.5	+ 2 27	6.7	120	11. 5. 17	I	19.1	−0.9	.....	.....	419	1	+0.25	A9	A–G	Globular cluster
6229*.....	16 44.2	+47 42	8.6	164.5	11. 10. 17	H	15.0	.....	.....	.....	415	1	+0.08	A3	A–G	Globular cluster
6341*.....	17 14.1	+43 15	6.2	60	3. 5. 18	W	15.7	+2.0	.....	.....	416	2	+0.11	A4	A–G	Globular cluster
				60	3. 5. 18	W	16.5	−1.7	.....	.....	417	1	+0.17	A6		
6543.....	17 58.6	+66 38	7.6	120	6. 10. 17	H	12.0	−0.9	.....	.....	.....	.....	.....	.....	.....	Planetary
				19	6. 10. 17	H	08.6	+1.8								
				5	6. 10. 17	H	412.2	−2.9	.....	.....	412	4	−0.05	P	P	
6720*.....	18 49.8	+32 54	8.9	90	25. 4. 17	I	394.0	0.0	.....	.....	.....	.....	.....	.....	.....	Planetary
					15. 10. 18	H	397.6	−3.9	397.8	−7.3	397	1	.....	P	P	Planetary
6826.....	19 42.1	+50 17	8.1	24	2. 10. 17	H	417.0	−2.4								
				16	2. 10. 17	H	14.6	+0.5	.....	.....	416	4	+0.11	P	P	Planetary
7009*.....	20 58.7	−11 46	7.2	45	5. 10. 17	H	12.6	+0.4	409.3	−0.3	411	4	−0.10	P	P	Planetary
7027.....	21 33	+41 50	8.5	49.3	24. 9. 17	H	11.5	−2.1	409.3	−0.5	411	4	−0.10	P	P	Planetary
7662.....	23 21.1	+41 59	7.6	51	24. 9. 17	H	18.5	−0.5								
				15	24. 9. 17	H	413.3	0.0	.....	.....	415	4	+0.08	P	P	Planetary

* See corresponding note at end of paper.

† I = Imperial; H = Hauff; W = Wellington.

objects with discontinuous spectra, but according to the results of Hertzsprung[1] as to the distribution of intensity in the spectrum of M 3 it seems that we must make this restriction also for the color-estimations of composite systems such as star-clusters and, perhaps, spiral nebulae. It seems that we must determine color and spectral type from the same spectral region in order to get the proper correspondence. However, an indication of a more extended validity for the computed color-indices of spirals and planetaries may be found in the very interesting results of F. H. Seares,[2] derived by comparison of ordinary photographs with those taken on isochromatic plates with yellow filter. The last three columns contain the spectrum calculated according to the principles set forth in the preceding, the spectrum observed by Fath and by Wolf, and the observed character of the object. The objects marked with an asterisk have a corresponding note at the end of the paper. There are also some notes for objects photographed which have not yielded satisfactory results.

The agreement between the observed spectra of spiral nebulae derived by Fath and by Wolf and those computed from the effective wave-lengths by applying the relations found for fixed stars is confirmed by the new results in this paper. According to an investigation of Lundmark, which will soon appear in print,[3] the probable parallax of the great nebula in Andromeda, the second object in Table III, is

$$\pi = 0''.0000057.$$

Because other spiral nebulae are probably at still greater distances, our results seem to confirm the opinion of Shapley that no sensible absorption exists in space. Even if we assume an improbable displacement of one interval in the spectral classification between the spectrum observed and computed, corresponding to $0^{M}.33$ in color-index, we should have the coefficient of absorption d, the change in color-index for one parsec:

$$d \leq 0^{M}.00000019.$$

[1] *Astrophysical Journal*, **41**, 10, 1915.

[2] *Proceedings of the National Academy of Sciences*, **2**, 553, 1916.

[3] Will appear in *Astronomische Nachrichten*, **209**, 369, 1919.

The list of possible objects for our instrumental means is not exhausted by the objects of Table III, and therefore investigations on the subject will be continued at the Observatory of Upsala.

NOTES

N.G.C. 221. Spiral[1] with very sharp nucleus. The accuracy of the measurements for this nebula cannot be estimated less than for an average fixed star. The four values of the effective wave-length obtained here for different intensities of image confirm our conclusion that the photographic Purkinje-effect is of quite subordinate importance for relatively "red" objects.

N.G.C. 224. The great nebula in Andromeda. By reason of the sharp nucleus tolerably distinct grating spectra were obtained, and consequently the value of λ_c is valid for the center of the nebula.

N.G.C. 598. On the plate of 11. 9. 17, exposure 180^m, there are very weak spectra of the most luminous part of the nebula (*Yerkes Publ.*, **2**, Plate XXV, $X = +34$ mm, $Y = -28$ mm), but the measurements are very uncertain and have not been adopted in Table III. The plate of 15. 11. 17, exposure 480^m, has a strong veil; the spectra are exceedingly faint.

N.G.C. 628. Though this nebula has a sharp nucleus, the time of exposure, 181^m, did not suffice to give a measurable image.

N.G.C. 650–651. We have not obtained measurable images with an exposure of 243^m.

N.G.C. 1068. According to Fath the spectrum of this nebula shows bright emission lines. The relatively low value of λ_c obtained here may possibly be explained by the assumption that the center of intensity in the photographic spectrum is displaced toward the value 413 $\mu\mu$, the mean value of λ_c for the planetary nebulae measured here. In the same way the low wave-lengths obtained for N.G.C. 2903 and 3627 may perhaps be caused by an appearance of bright lines in their spectra.

N.G.C. 1514. This nebula is a very remarkable object. It was classified by Herschel as a planetary, but on Mr. Isaac Roberts' photographs[2] it has been found to have spiral structure. The value of λ_c shows that very probably we have to do with a planetary nebula.

N.G.C. 2245, 2261, 2245 resembles a star and shows nebulous rays. 2261 is Hubble's variable nebula.[3] The wave-lengths measured are in both cases valid for the nuclei of the objects.

N.G.C. 2403. This nebula has small surface intensity and resembles N.G.C. 598 as to its structure. The time of exposure, 195^m, is too small to give measurable images.

[1] A. R. Hinks, *Monthly Notices*, **71**, 588, 1911.

[2] *Monthly Notices*, **74**, 234, 1914. [3] *Astrophysical Journal*, **44**, 190, 1916.

N.G.C. 2841. The measures on the plate taken with objective I are very sure. Because there is an irregularity in the image given by objective II and, in contrast to what is usually the case, the wave-length falls out lower than on the plate of objective I, it has not been used in deriving the definitive value of λ_c.

N.G.C. 2903, 2905. The nebula 2905, which is a condensation in one of the spiral arms of 2903, is not measurable. A somewhat longer exposure, however, would certainly make it possible to determine its wave-length and thus to derive the difference in color between the two objects.

N.G.C. 2976. This nebula, which consists of a round nebulous patch with small surface intensity, is found on the plates containing 3031, 3034, and 3037, but only the central image is perceptible. The total intensity is estimated by Kritzinger to $10^{M}.0$ and by Holetschek to $10^{M}.7$.

N.G.C. 3034. On account of an error of reduction in our first paper the wave-length of the nebula had been given to be 415 $\mu\mu$ instead of 425 $\mu\mu$, which is the right value according to the measurements. However those measurements were rather uncertain, and the value here given must be considered better, though it is subject to some uncertainty due to the faintness and irregular form of the object.

N.G.C. 3077. The difference between our new and old determinations is considerable. None of the measurements are very sure, but a greater weight ought to be ascribed to the value from our first paper.

N.G.C. 3377, 3379, 3384, 3412. These nebulae are reproduced on the Wolf-Palisa chart No. 109. Though it is impossible to discern their structure plainly, they seem to be small spirals, which also follows from our values of their effective wave-lengths. See also *Mount Wilson Report for 1917*, p. 214.

N.G.C. 3587.[1] If we refer the central star to type O, which is a manner of interpreting the low effective wave-length, and assume its absolute magnitude to be $-1^{M}.0$, according to the mean value for O-type stars in Kapteyn's, Hertzsprung's, Gyllenberg's, and van Maanen's investigations, we get the parallax $\pi_{M97} = 0''.0001$. If we make the same assumption for the object: R.A. $11^{h}9^{m}.2$, Decl. $+55°31'$, which is a star connected to nebulous masses, we get a hypothetical parallax of the same order of magnitude. The similarity of the appearance of the latter object[2] with the present appearance of Nova Persei[3] makes it possible that the nebulous star may be a late nova, and thus probably a star with the characteristics of type O.

N.G.C. 3623, 3627. Spirals with well-exposed spectra. The difference in effective wave-length obtained by us is confirmed by the circumstance that while according to Holetschek 3627 is $0^{M}.3$ brighter visually than 3623, the difference in photographic magnitude is much greater. Thus we find from

[1] *Monthly Notices*, **67**, 543, 1907.

[2] See Ritchey, *Astrophysical Journal*, **32**, 26, 1910; *Mt. Wilson Contr.*, **2**, 283, 1910, Plate XVII.

[3] *Publications of the Astronomical Society of the Pacific*, **30**, 163, 1918.

estimations performed by means of the grating spectra the photographic difference

$$M_{3623} - M_{3627} = +0^M7$$

and consequently a color-index about $\frac{1}{2}^M$, which is in tolerably good agreement with the result 0^M7 obtained from the color-indices in Table III.

N.G.C. 4736. The first and second values in the table are taken from our first paper and are valid for the nebula in its entirety and the central nucleus respectively. For the new images measured such a discrimination has not been possible.

N.G.C. 6205. The short exposures of this cluster give too faint and diffuse images of the condensations measurable in our previous paper. For the long exposure, 120^m, spectra and central image of the entire cluster begin to overlap.

N.G.C. 6229. This object has sometimes been classified as a planetary nebula, sometimes as a globular cluster. Shapley's investigations have shown that the object is a dense globular cluster containing about 1500 stars. The grating spectra on our plate are very weak and the measurements rather uncertain.

N.G.C. 6341. This cluster has given rather distinct and easily measurable spectra. The first value in Table III refers to the cluster in its entirety, the second to a conspicuous condensation situated in the northern edge of the cluster.

N.G.C. 6503. Irregular nebula. Not measurable on our plate, exposed 120^m.

N.G.C. 6720. Though our measurements on different plates give rather concordant results, they must be considered very uncertain. It is evident, however, that the nebula gives a lower wave-length than any other measured by us. The stars *c* and *d* mentioned by Burns,[1] with the spectral types A and F respectively, are measurable and gave the accordant wave-lengths 417.5 $\mu\mu$ and 421.4 $\mu\mu$.

N.G.C. 6946. On account of an erroneous statement as to the magnitude of Nova Ritchey, 1917, this nebula was exposed 300 minutes. The nebula has very small surface intensity and seems to be of the same type of spirals as N.G.C. 598 and 2403. Not the faintest grating spectra are to be found on our plate.

N.G.C. 7009. The great value of the zenith-correction makes the value of λ_c somewhat uncertain.

N.G.C. 7023. This irregular nebula envelops the star[2] B.D. 67°1283. On our plate there is a faint trace of the nebula but not sufficiently strong for a determination of λ_c. The star is overexposed, and therefore we have still no value of its effective wave-length.

N.G.C. 7814. With an exposure of 210 minutes the spectra are too weak to be measured with any accuracy.

[1] *Lick Observatory Bulletin*, No. 193, 1911.

[2] *Astronomische Nachrichten*, **204**, 41, 1917.

SUMMARY

Photographic effective wave-lengths of nebulae and clusters, in all 44 objects, have been obtained, whereby it has become evident that different kinds of objects correspond to certain characteristic intervals of wave-length. Thus the planetary nebulae have wave-lengths below 416 $\mu\mu$, and the mean value for six objects, excluding the Owl nebula and the Ring nebula in Lyra, is 413 $\mu\mu$. The four globular clusters investigated have wave-lengths ranging between 415 $\mu\mu$ and 425 $\mu\mu$ and the mean value is 419 $\mu\mu$, corresponding to the spectral type A8. Fourteen nebulae known as spirals give the mean value 429 $\mu\mu$, spectral type G2; the values of λ_c range between 420 $\mu\mu$ and 436 $\mu\mu$. Several objects in Table III may with great probability be referred to the spiral class by considering the values of their effective wave-lengths.

From the effective wave-lengths we have computed spectral types and color-indices, using the relations found for fixed stars. For clusters and spiral nebulae the convertion of λ_c into a spectral type of the usual sequence B–M seems to be legitimate; on the other hand we have seen that a low value of λ_c for a nebulous object indicates with a fair probability the character of a planetary nebula, a fact which may be expressed by assigning the nebular spectral class P to such objects.

From the facts mentioned we may draw the conclusion that at least a rigorous discrimination between planetary and spiral nebulae may easily be effected by measuring photographic effective wave-lengths, a circumstance which may be of importance for the future classification of faint nebulae.

ASTRONOMICAL OBSERVATORY
UPSALA
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THE ORBIT OF THE SPECTROSCOPIC BINARY BOSS 2285[1]

By ALFRED H. JOY AND GIORGIO ABETTI

The variable velocity of the star Boss 2285 = Pi 8[h]108 ($\alpha = 8^h30^m.5$, $\delta = +6°58'$; 1900.0) was discovered at this observatory from plates taken in 1918.[2] With Boss 2286 = Lal. 16895 it forms the physical system β G.C. 4677 = Σ 1245. The measures of position angle and distance show no change since the time of the observations by Dembowski. The lack of relative motion is confirmed by van Maanen from measures of the two components on his parallax plates which give: 1916.23, P.A.25°.0, dist. 10".49. The Harvard apparent magnitudes, the proper motions by Boss, the Mount Wilson spectral types, and the absolute parallaxes with corresponding absolute magnitudes by van Maanen are:

	Harvard M	μ	Spec.	π	M
Boss 2285 . . .	6.04	0".195	F7	+0".022	+2.8
Boss 2286 . . .	7.15	0.201	G3	0.026	4.2

The absolute magnitudes determined by Adams and Joy from the spectra are 4.5 and 5.5, which would give a mean parallax of 0".048. Measures of the four plates of Boss 2286 give radial velocities ranging from +23.4 km to +31.0 km with a mean of +27.9 km which may be compared with the velocity of +25.2 km found in this investigation for the center of mass of the system Boss 2285. The spectrum of the secondary star is not visible on any of the plates.

Table I gives a list of the photographs. The Greenwich mean time refers to the middle of the exposure; the phase is derived from the period 14.296 days and refers to the epoch J.D. 2421598.0. The plates have been taken with the 60-inch reflector and the 18-inch single-prism spectrograph. The various plates have been

[1] *Contributions from the Mount Wilson Observatory*, No. 172.

[2] *Publ. Astr. Soc. of the Pacific*, 31, 41, 1919.

measured by Adams, Joy, Abetti, Strömberg, and Miss Burwell, and the number in the table indicates the number of measures of each plate. No systematic difference between the measures was found, so that the mean result has been used in the table.

TABLE I

Plate No.	Date	G.M.T.	Phase	Velocity	No. of Meas.	O−C	Remarks
				km		km	
7853.....	1919, Feb. 8	18^h40^m	$0^d.490$	− 0.4	3	+0.1	
5770.....	1917, May 6	15 42	0.686	− 2.0	4	+1.8	
6818.....	1918, Mar. 31	17 41	0.961	− 3.3	4	−2.1	
6876.....	1918, Apr. 29	16 51	1.334	+ 3.3	3	+1.6	
7936.....	1919, Mar. 10	18 26	1.888	+ 6.5	3	−2.6	
6886.....	1918, Apr. 30	17 22	2.356	+16.5	3	−0.1	
7943.....	1919, Mar. 11	17 25	2.846	+25.9	3	+1.6	
7944.....	1919, Mar. 11	17 52	2.864	+25.0	3	+0.5	
6575.....	1918, Jan. 21	21 22	3.594	+35.4	3	+2.0	
6589.....	1918, Jan. 22	20 41	4.565	+38.5	3	−1.7	
7864.....	1919, Feb. 12	20 55	4.584	+40.7	3	+0.4	
7772.....	1919, Jan. 15	21 29	5.199	+47.9	4	.....	Weak
6597.....	1918, Jan. 23	21 16	5.590	+41.7	5	−1.3	
7786.....	1919, Jan. 17	22 28	7.240	+44.5	4	+2.6	
6504.....	1917, Dec. 29	21 40	9.199	+32.8	4	−2.5	
6615.....	1918, Jan. 27	20 17	9.549	+35.4	3	+1.8	
7975.....	1919, Mar. 18	16 03	9.789	+29.9	3	−2.5	
6515.....	1917, Dec. 30	23 47	10.287	+32.2	4	+2.5	
6925.....	1918, May 23	15 59	11.002	+16.9	3	.....	7-in. camera
7914.....	1919, Feb. 19	20 17	11.557	+20.3	4	−1.0	
6659.....	1918, Jan. 30	19 10	12.503	+15.2	3	+1.5	
6799.....	1918, Mar. 29	17 29	13.248	+ 6.7	4	−0.6	
6744.....	1918, Mar. 15	19 30	13.628	+ 2.9	4	−1.1	

The velocity-curve was first drawn through all the observations with the aid of a preliminary period determined from a comparison of the times of recurrence of approximately equal velocities. Since the eccentricity was shown to be considerable, the method of Lehmann-Filhés was used for determining the approximate elements of the orbit, which were then slightly adjusted so as to make the residuals as small as possible. Plate 7772, which was very weak, and 6925, which was taken with the 7-inch camera, showed rather large differences and were omitted in the final solution and are not shown in the figure. A least-squares solution, with twenty-one equations of condition of the form given by Lehmann-Filhés and Schlesinger, was used for improving the approximate elements. The following corrections were found:

$\delta P = +0.005$ days
$\delta K = -0.26$ km
$\delta\omega = -13.^{\circ}00$
$\delta e = +0.006$
$\delta T = -0.526$ days
$\delta\gamma = +0.03$ km

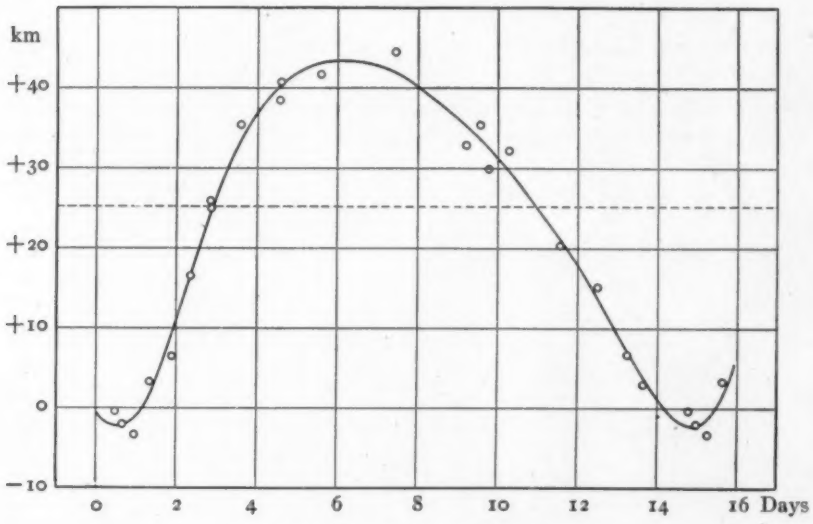

FIG. 1.—Velocity-curve of Boss 2285

After applying these corrections, we have for the final elements:

$P = 14.296 \pm 0.003$ days
$K = 22.74 \pm 1.20$ km
$\omega = 220.^{\circ}80 \pm 1.^{\circ}98$
$e = 0.276 \pm 0.021$
$T =$ J.D. $2421599.474 \pm 0.157 = 1918$, Jan. 5, $11^{h}22^{m}$ G.M.T.
$\gamma = +24.23 \pm 0.62$ km
$a \sin i = 4{,}300{,}000$ km
$\dfrac{m_1^3 \sin^3 i}{(m+m_1)^2} = 0.015\ \odot$

The residuals O–C given in Table I were computed with these elements. The computed curve and the observed velocities are shown in Fig. 1.

MOUNT WILSON OBSERVATORY
April 1919

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